Multiple Solution Methods for Teaching Science in the Classroom

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Improving Quantitative Problem Solving using Dimensional Analysis and Proportional Reasoning

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Universal Publishers
Boca Raton, Florida

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Universal Publishers
Boca Raton, Florida • USA
2008

ISBN-10: 1-59942-988-8
ISBN-13: 978-1-59942-988-5

www.universal-publishers.com

Library of Congress Cataloging-in-Publication Data

DeMeo, Stephen, 1961-
Multiple solution methods for teaching science in the classroom : improving quantitative problem solving using dimensional analysis and proportional reasoning / Stephen DeMeo.
p. cm.
Includes bibliographical references.
ISBN 1-59942-988-8 (pbk. : alk. paper)
1. Science--Study and teaching. 2. Science--Methodology.
3. Reasoning. 4. Problem solving. I. Title.
Q181.D368 2008
507.1--dc22
2008014365

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Acknowledgments

To those who will read my book I enthusiastically thank you and I hope it challenges or supports your notions of how science can be taught. To those who act on my book I wish you good speed in using, personalizing, and elevating those ideas that you find most valuable.

I would like to extend my thanks and gratitude to: Hunter College of the City University of New York for providing a sabbatical to complete this work; my wife Pam and daughter Sophia who had to put with the often isolated, grumpy writer syndrome that plagues many of us academics; my mentor Jean Lythcott who has had such a profound impact on my understanding of education; my friends and colleagues who were kind enough to comment on this work and offer timely advice; and last but not least, the numerous students who trusted me and became involved in studies that make up a good portion of this book. Once again, my deepest thanks.

CHAPTER 1

Stepping into the Arena

It is widely known that problem solving is a central subject and fundamental ability in the teaching and learning of science. Problem solving is of tantamount importance to science because it elucidates the means as well as identifies conceptions that are invaluable to understanding the physical world. Furthermore, problem solving underscores the value our society places on scientists and engineers to improve the quality of life and to profitably transform ideas into products. This book is the first of its kind to explore multiple solution methods to quantitative problems in science education. While researchers rightly have paid much more attention to learners' conceptual understanding of content, it is vital that methodological or procedural knowledge be critically examined. One reason for doing this is because scientific methods activate concepts – they put ideas into use – in order for the physical world to be controlled and understood.

To date, research on the use of multiple solution methods has been conducted in mathematics education, but is woefully underdeveloped in the area of science. A book describing studies in science education that pertains to multiple methods is needed and would initiate further inquiry into this promising area of problem solving.

A major focus of the book will be on two widely used methods, Dimensional Analysis (DA) and Proportional Reasoning (PR). These specific methods have been chosen for a number of important reasons. Both can be used to solve a large number of problems, they are applicable to all of the four academic sciences (biology, chemistry, physics, and earth science), they are taught in varying degrees in middle school, high school and colleges, and lastly, the methods are used in schools internationally.

While PR has been strongly emphasized in the U.S. mathematics curriculum for many years now, it is commonly used in earth science and chemistry courses. The DA method, which is seldom used in math courses, is used extensively in chemistry and physics. While many readers might have experience with DA and PR either from their earlier years as students or later as teachers, I will not make the assumption that everyone has previous knowledge of both. Therefore I will define each method succinctly and provide an example to illustrate their differences.

Dimensional Analysis

DA is also known as Unit Analysis as well as the Factor Label Method. The key concept behind DA is the dimension of a quantity or unit. The unit label serves to guide the set-up of the calculation used to solve the problem. In the example below involving the conversion of milliliters to liters, the step-wise process can be seen. First the unit of the quantity must be determined, in this case milliliters, then the given amount and the conversion factor must be arranged so that all units cancel out with the exception of the one that is desired in the solution. The unit in the example is liters and so the conversion factor must be expressed in a manner that allows for the cancellation of the mL units and the isolation of the L units. The numerator and denominator represented in the conversion factor are expressed as a ratio signifying equivalence not equality.

Problem: How many liters are equal to 125 milliliters given that 1000mL = 1L?

Solution using DA:

Step 1: $\mathrm{X\,L} = (125\ \mathrm{mL})\left(\frac{1\ \mathrm{L}}{1000\ \mathrm{mL}}\right)$

Step 2: $\mathrm{X\,L} = 0.125\ \mathrm{L}$

The essence of DA is found in matching units: the beginning of the calculation − the right side before the equal sign − must eventually have the same unit as the left side. If the units do not match, then the student knows there is an error in the problem set-up.

Proportional Reasoning

With PR the focus is not on the unit. Rather, the quantities involved and the relationship between quantities are more important. These relationships are usually expressed in the conversion factor or ratio between two quantities. A proportion requires two sets of ratios, one being a scaled up or scaled down version of the initial ratio. A proportion represented as a/b=c/d simply can be read as "a is to b, as c is to d". In most scientific fields "a" and "c" commonly refer to the same quantity having the same units; this also applies to "b" and "d". Symbolically, it would be more appropriate to write $a/b=a^1/b^1$, as in the example 1ft/12in. = 2ft/24in.

When two ratios are in direct proportion to each other, the change from "a" and "b" to "a^1" and "b^1" increases. If an indirect relationship exists, then "a^1" and "b^1" decrease. When two ratios are made equal to each other, one of the quantities can serve as an unknown variable. The process used to solve for the unknown variable, usually represented by the letter "X", is cross multiplication. Below is the same problem solved with PR. Notice that while there are additional steps, the math that is required only involves multiplication and division.

Solution using PR:

Step 1: $$\frac{1000\text{ mL}}{1\text{ L}} = \frac{125\text{ mL}}{X\text{ L}}$$

Step 2: Cross multiplying gives,

(X L)(1000 mL) = (1 L)(125 mL)

Step 3: X is isolated on one side by dividing each side by 1000mL:

(X L)(1000 mL)/(1000 mL) = (1 L)(125 mL)/(1000 mL)

Step 4: X = 0.125 L

The algorithmic steps involved in each method are straightforward and require little math beyond basic manipulations. These characteristics are what make both these methods so useful to teachers of introductory science courses: they are reliable, easy to use, understandable, and applicable to all four science disciplines that compose a secondary and introductory college science curriculum.

One example for which DA and PR cannot be used is the conversion between degrees Celsius and Kelvins. The conversion between these two scales is done by a constant difference rather than multiplying the given quantity by a constant ratio or conversion factor.

As multiple methods, DA and PR represent two ways to produce a single solution since data given in the problem is transformed in different ways to produce a single answer. The idea of multiple methods can also refer to two different procedures, generating different data sets that also lead to a single solution. For example, there are a few different ways to measure the density of a piece of metal. Different procedures are used but a single answer, lying within a calculated level of uncertainty, is understood as being correct. Therefore, when multiple methods or multiple solution methods are mentioned in the text, I am referring to both approaches, data that is transformed mathematically as well as different activities that involve more than one procedure. The emphasis of this book will be the use of multiple methods in the classroom as opposed to the laboratory. Now that the basic subject matter of this book has been described, the more specific organizational structure of the chapters can be discussed.

Organizational Structure

This book is conceptually divided into two parts. The first section, which consists of the first five chapters, begins by examining the extent to which DA and PR are present in the science curriculum. High school and introductory college textbooks written for students taking courses in earth science, biology, chemistry and physics are reviewed and example problems are identified which draw upon DA and PR to provide solutions. The main reason for doing this is to find out the degree to which these two methods are used in high school and first year college science courses, and to suggest a way these processes can become integrated. Because of the very fundamental and generalizable nature of these problem solving methods, there is great potential to utilize them not just in one class but through all the four sciences and at different grade levels.

One of the chemistry problems that can be solved by both DA and PR methods involves limiting reactants, a topic usually found in the unit on stoichiometry. The word "stoichiometry," refers to quantitative aspects of chemistry, and limiting reactant problems usually involve determining the amount of product from a given amount of two or more reactants. Suffice it to say that limiting reactant problems represent one of the most complex problems that introductory students encounter. In Chapter 3, I use this type of problem in order to reveal what teachers think about DA and PR. A complicated problem such as this provides teachers with a context to articulate significant differences between the two methods. A national survey was conducted to determine which method was preferred by high school and college chemistry teachers. The 55% return rate and the comments from open-ended questions indicated that many teachers feel strongly about the method they use to solve limiting reactant problems. The survey, when analyzed, has led to the identification of a preferred method, and has introduced for discussion how these methods should be represented in introductory science textbooks.

Chapter 4 looks at what researchers say about DA and PR first in terms of Piaget's developmental ideas of learning then in regard to understanding conceptual knowledge. In this chapter, the focus is on how well students who use DA and PR methods understand the meaning behind their solutions. The focus on a learner's knowledge challenges the assumption that students who correctly solve problems using algorithms do so with understanding. Improving student understanding can be done in a variety of ways. For instance, concept mapping, Vee diagramming, and cooperative learning have promoted student understanding of a wide variety of concepts. These instructional strategies and others are introduced and discussed. I conclude this chapter by arguing which method DA or PR would be the best method to use when solving stoichiometry problems.

Chapter 5 continues the thread of instructional techniques by examining how DA and PR can be taught in a classroom. Since many of the teachers participating in the survey mentioned they have used visual organizers to teach students how to solve stoichiometry problems, I thought it timely to present a study involving the construction of a stoichiometry map that incorporates conceptual and methodological knowledge. This study follows science and pre-medical students as well as nursing majors during a map making workshop. Quantitative as well as qualitative data on their perceptions of achievement are collected. While there are numerous stoichiometry maps cited in the literature there is very little quantitative research conducted on their effectiveness. This study provides a much needed analytical evaluation.

Chapter 6 is the start of the second half of the book and deals with the issue of choosing between multiple methods. Once again there is little research being conducted on this topic in the area of science education. The lack of interest in multiple methods is mainly because teachers assume that students are easily confused when faced with different ways to solve a problem. To address this point, the teacher survey is revisited and compared to a study I conducted with students faced with making a choice between DA and PR. My goal is simply to determine if choice impedes students' ability to solve problems. My investigation of this question leads to a presentation of numerous other methods in addition to DA and PR that can be used to solve limiting reactant stoichiometry problems.

With an emphasis on the research literature, Chapter 7 brings together current studies on the use of multiple methods in science education. While not many in number, they are enlightening. For example, one researcher presented multiple methods to students after they have worked on solving a physics problem. A review and comparison session allowed learners to reflect on their work and articulate their understanding. When an exam on this topic was given months later scores revealed a significant increase in achievement for these students in the subject area. This study as well as others will help to answer an important question: Would it be beneficial for science students to understand multiple ways of solving problems, when methods are taught side by side? I will try to answer this question by first examining multiple methods in mathematics education, and then by looking at related topics such as multiple representations, inquiry-based laboratory activities, open-ended problem solving, and critical thinking. I will address the issue that additional methods prohibit a teacher from adequately "covering content". By doing this, multiple solution methods will be placed in the larger context of a learner-center pedagogy.

I conclude this book by making some unpopular recommendations, one being about the need to integrate DA and PR, rather than viewing them as oppositional methods. A second recommendation is to use multiple methods like DA and PR throughout the science curriculum from late elementary school, through middle and high school, into the introductory college level. This systemic view also takes into consideration our analytical partners, the many math teachers who currently teach proportional reasoning to students taking science courses. I expect a certain amount of debate over these ideas because they represent a challenge to the existing single-method paradigm. Hopefully my argument will be persuasive and create some advocates for these curriculum changes.

From my work as well as from others, we know that the influence a teacher has on learners can be very powerful. So too is the influence of a textbook on the teacher. The changes that I am arguing for are not just relevant to teachers but they also pertain to authors of our introductory science textbooks. We cannot assume that textbooks are written with the needs of teachers or learners in mind, or from the findings supported by research. Books like the one you are reading now are necessary if only for the reason to propel teachers, authors, and others in the educational community to consider alternative ways of doing things.

At the end of the second chapter I changed my focus from a wide perspective to a much narrower one. I stopped discussing all of the four academic sciences and centered much of my attention on chemistry. This was not an accident since I am a chemist by training and enjoy discussing the nuances of the subject. However, my penchant for chemistry does not limit the usefulness of this book to educators from other scientific backgrounds. The map making workshop presented in Chapter 5 can be generalized to other subject areas and adapted by teachers to accommodate the needs of their students. Furthermore, the two chapters surveying the research literature include studies that involve all of the sciences. Even my work on student choice would be appropriate to any science teacher who considers giving students more freedom in their problem solving tasks.

The content and style of this text are purposely written for science educators who work on the late elementary to the introductory college levels. The text can be used by pre- and in-service teachers, as well as instructors of teacher education preparation courses. After listening to science students complain that educational texts are too "soft" or "abstract", and that many lack data and conclusions supported by evidence, I have written a book that squarely rests on the enumeration of empirical studies. I included numerous data tables, graphs, diagrams, a thick appendix, and of course many current references. Another reason for including empirical studies is to call attention to the need for scholarship in this area. My few studies are a beginning; much more work is needed to elucidate this area. With the presence of data driven work, this book is less a narrative and more akin to reading a series of extended science articles. While this might lend itself to uneven reading, the style should feel familiar to readers with a science and math background.

Another uncommon characteristic this book eschews involves quotations. I included numerous quotes from teachers as well as students mainly because the studies I conducted asked many open-ended questions. In addition to organizing and tallying comments, I wanted to capture the voice of the many teachers and students who contributed their opinions to this project. Without them, I strongly believe the text would be less meaningful.

While this book is research-based as well as concerned with the literature, it is also sensitive to the varied background of teachers who might want to read this text. In my opinion this book is within the conceptual grasp of many if not all science educators on at least two accounts. The book centers on two fundamental, easily understood methods and refrains from sophisticated statistics that can retard the understanding of quantitative measurements. This book brings abstraction down to the level of details, but in a manner that does not over conflate the subject matter or research process.

The focus of the book has also been considered. As a teacher of science education on the graduate level, I am in favor of using multiple texts on specific topics rather than requiring students to read one or two general textbooks. I like the depth an author can achieve when focused on an important topic. Furthermore, in terms of time I prefer to devote one week to a topic-based text, as opposed to spending most of the semester anchored to a larger book. Unfortunately, there are not enough of these specialized books available to teachers of science education courses.

Having said what this book is about, I must mention a few topics that are not included in the text. This book does not describe the best ways to teach students DA and PR. Nor does it describe the misconceptions that students bring to the classroom or that are formed as a consequence of poor instruction. The literature in this area is already robust. This text is not a survey of numerous multiple methods that are used or can be used in the four academic sciences. Instead, this book relies on quantitative empirical studies, reviews of the literature, a case study focusing on DA and PR, and the content area of stoichiometry to elucidate how multiple methods can help students improve problem solving in science education. I have written it with the general intention of introducing new and experienced teachers to an alternative way of teaching science, one that can potentially benefit their students' ability to solve problems and understand concepts.

CHAPTER 2

Islands of Knowledge

"Dimensional Analysis and Proportional Reasoning are places that the math-science relationship needs to improve if we are to effectively teach our students".

(teacher survey coded #360)

Guiding Question

How and to what extent are DA and PR used in the current high school and introductory college science curriculum?

DA and PR have been used in the high school and college curriculums for many years. But their use over the course of a secondary science curriculum has been sporadic, isolated, and non-complimentary. For example, while PR is taught in the later elementary grades, it is rarely found in biology or physics textbooks as a method to solve problems. DA on the other hand is only first introduced in high school chemistry or physics courses and is seldom used elsewhere. These two methods are rarely paired in textbooks in any of the subject areas.

The problems that lend themselves to DA and PR vary from one science discipline to another. In this chapter I will look at how authors of introductory science textbooks use DA and PR to solve an assortment of problems in the four sciences. I will present problems in each discipline and make recommendations on how DA and PR can be integrated throughout the sciences.

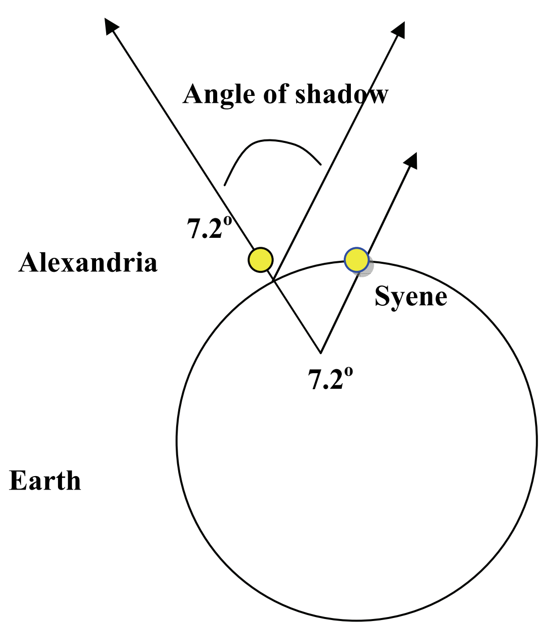
Earth Science

After reviewing many high school earth science textbooks and some older, introductory college astronomy texts I found no examples of problems being solved using DA. PR, on the other hand, is used repeatedly in these texts to determine the circumference of the earth or what is referred to as Eratosthenes measurement (Frankonoi, Morrison, & Wolff, 1997). An example of this type of problem is given below. It involves proportionally comparing distances and angles between two cities with the earth itself.

1) Circumference of the Earth

Problem: Knowing that the distance from Alexandria to Syene is 787 km, calculate the circumference of the earth using the data found in the below diagram.

On solstice, angle of inclination or shadow is 7.2^{o}



a) Solution using Proportional Reasoning

$$\frac{787 \text{ km}}{7.2 \text{ degrees}} = \frac{\text{X Earth's Circum. in km}}{360 \text{ degrees}}$$

X Earth's Circum. = 39350 km = 39000 km

b) Solution using Dimensional Analysis

$$\text{X Earth's Circum. km} = \frac{787 \text{ km}}{7.2 \text{ degrees}} \times 360 \text{ degrees}$$

X Earth's Circum. = 39350 km = 39000 km

There are a few other areas in earth science textbooks that draw upon PR. For instance, topographic maps and problems dealing with the composition of air are quite amenable to this method. The use of PR in topographic maps usually involves the issue of scale as it relates to a geographical model. The problem below is solved using both PR and DA.

The DA solution to the topographic problem can be challenging for learners because it is difficult to identify the units involved. Problems involving the percent composition of gases in air also have a similar issue. The solver using DA must know that percent is unitless. These observations will be made clear as one reads through the problems.

2) Scale of Topographic Maps

Problem: A map has a ratio scale of 1:24,000 (1 inch is equivalent to 24,000 inches). How many miles per inch is this equivalent to?

a) Solution using Proportional Reasoning

24,000 inches must be converted to miles using the conversion, 5280 feet = 1 mile

$$\frac{12 \text{ inches}}{1 \text{ foot}} = \frac{24{,}000 \text{ inches}}{X \text{ feet}} \qquad X = 2000 \text{ feet}$$

$$\frac{5280 \text{ feet}}{1 \text{ mile}} = \frac{2000 \text{ feet}}{X \text{ miles}} \qquad X = 0.38 \text{ miles}$$

b) Solution using Dimensional Analysis

$$X \text{ miles/inch} = \frac{1 \text{ foot}}{12 \text{ inches}} \times \frac{1 \text{ mile}}{5280 \text{ feet}} \times \frac{1 \text{ inch}}{24{,}000 \text{ inches}}$$

$$X \text{ miles/inch} = 6.6 \times 10^{-10}$$

The above value is *incorrect* because the scale 1:24,000 has the same units (inches) in the numerator and the denominator. Students who primarily focus on the manipulation of units could easily make an error. The correct set-up could be found if one evaluated the answer and concluded that it was nonsensical. Below is the correct set-up and answer.

$$X \text{ miles/inch} = \frac{1 \text{ foot}}{12 \text{ inches}} \times \frac{1 \text{ mile}}{5280 \text{ feet}} \times \frac{24{,}000 \text{ inches}}{1 \text{ inch}}$$

$$X \text{ miles/inch} = 0.38 \text{ miles/inch}$$

3) Composition of Air

Problem: A 5 L outdoor air sample was taken. If air consists of about 78% nitrogen, 21% oxygen, and 1% Argon and other gases, then what is the approximate volume in liters of oxygen in the sample?

a) Solution using Proportional Reasoning

To solve this problem one must understand percent refers to a certain number of parts over 100 parts.

$$\frac{21}{100} = \frac{X \text{ liters}}{5 \text{ liters}}$$

X = 1.05 = 1 liter

b) Solution using Dimensional Analysis

Because percent is unitless, dimensional analysis must be used thoughtfully. Twenty-one, not 100, must be used as the numerator.

$$\text{Volume liters} = \frac{21 \text{ liters}}{100 \text{ liters}} \text{ x } 5 \text{ liters}$$

Volume liters = 1 liter

Biology

After reviewing introductory biology textbooks on the high school and college levels published over a span of 30 years, I did not find any consistent use of DA and PR methodologies to solve problems. This absence is due most likely to the descriptive character of introductory biology and the fact that mathematics is not an integral part of the curriculum. While this is so, there are places that DA and PR can be used in the biology curriculum. For instance, the areas of microscopy, cell division and blood circulation can serve as examples of topics where DA and PR methods can be used to solve quantitative problems.

1) Measurement used in Microscopy

Problem: Most eukaryotic cells are 10 to 30 micrometers in diameter. How many centimeters long is a 10 micrometer cell? (1um = 1micrometer = 1 micron = 1/10,000 cm = 0.0001 cm)

a) Solution using Proportional reasoning:

$$\frac{1 \text{ um}}{0.0001 \text{ cm}} = \frac{10 \text{ um}}{X \text{ cm}}$$

X = 0.001 cm

b) Solution using Dimensional Analysis

$$X\text{ cm} = 10\text{ um} \times \frac{0.0001\text{ cm}}{1\text{ um}}$$

$$X\text{ cm} = 0.001\text{ cm}$$

2) Cell Division

Problem: About 2 trillion cell divisions occur in an adult every 24 hours. How many occur in 1 second?

a) Solution using Proportional Reasoning

$$\frac{1\text{ hour}}{60\text{ minutes}} = \frac{24\text{ hours}}{X\text{ minutes}} \qquad X = 1440\text{ minutes}$$

$$\frac{60\text{ seconds}}{1\text{ minute}} = \frac{X\text{ seconds}}{1440\text{ minutes}} \qquad X = 86400\text{ seconds}$$

$$\frac{2x10^{12}\text{ divisions}}{86400\text{ seconds}} = \frac{X\text{ divisions}}{1\text{ second}} \qquad X = 2x10^{7}\text{ div/second}$$

b) Solution using Dimensional Analysis

$$X\text{ div./second} = \frac{1\text{ minute}}{60\text{ seconds}} \times \frac{1\text{ hour}}{60\text{ minutes}} \times \frac{2x10^{12}\text{ divisions}}{24\text{ hours}}$$

$$X\text{ div/second} = 2x10^{7}\text{ div/second}$$

3) Circulation

Problem: How many minutes does it take to circulate your blood if you have 5 liters in your body and your heart ejects about 0.07 liters per beat. You will need to find your pulse.

a) Solution using Proportional Reasoning

Measured pulse: 80 beats per minute

$$\frac{0.07 \text{ liters}}{1 \text{ beat}} = \frac{5 \text{ liters}}{\text{X beats}} \qquad \text{X} = 71.43 \text{ beats}$$

$$\frac{80 \text{ beats}}{1 \text{ minute}} = \frac{71.43 \text{ beats}}{\text{X minutes}} \qquad \text{X} = 0.893 = 0.9 \text{ minutes}$$

b) Solution using Dimensional Analysis

$$\text{X minutes} = \frac{1 \text{ beat}}{0.07 \text{ liters}} \times \frac{1 \text{ minute}}{80 \text{ beats}} \times 5 \text{ liters}$$

$$\text{X minutes} = 0.9 \text{ minutes}$$

Chemistry

In chemistry, DA and PR methodologies are much more common. This is mainly because chemistry draws upon algebra to a greater extent than biology and earth science to solve problems. Like the previous biology example involving unit conversions, chemistry often makes use of simple conversions when dealing with mass. Conversions from grams to kilograms are often commonplace, as is the conversion from grams to moles. This latter conversion used in the example below uses the molar mass of elements taken from the Periodic Table.

1) Grams to Moles Conversion

Problem: How many moles are in 100.0 grams of zinc oxide (ZnO)?

Pre-requisite tabulation of the molar mass of ZnO
65.39 g/mol Zn + 16.00 g/mol O = 81.39 g/mol ZnO