

Fundamental Concepts in Electrical and Computer Engineering with Practical Design Problems

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Preface

Purpose

In many cases, the beginning engineering student is thrown into upper-level engineering courses without an adequate introduction to the basic material. This, at best, causes undue stress on the student as they feel unprepared when faced with unfamiliar material, and at worst, results in students dropping out of the program or changing majors when they discover that their chosen field of engineering is not what they thought it was. The purpose of this text is to introduce the student to a general cross-section of the field of electrical and computer engineering. The text is aimed at incoming freshmen, and as such, assumes that the reader has a limited to nonexistent background in electrical engineering and knowledge of no more than pre-calculus in the field of mathematics. By exposing students to these fields at an introductory level, early in their studies, they will have both a better idea of what to expect in later classes and a good foundation of knowledge upon which to build.

Organization

This text consists of twenty chapters organized into five distinct units. In the first unit, the student is presented with a review of some key mathematical concepts. This overview is designed to fill in any gaps in the student's previous mathematical background and to provide a foundation for their study of the remaining chapters of the text. The first unit consists of three chapters. Each of these chapters focuses on a different important area of mathematics with which the beginning engineering student should be acquainted. The topics, covered in the first three chapters of the text, include, numeric bases, complex numbers, and basic elements of vectors and matrices.

Chapter 1 introduces the student to the concept of numerical bases. They will learn what a base is, how to convert back and forth between a given base and decimal, and how to work with some of the most common numerical bases, including binary, octal, and hexadecimal.

Chapter 2 covers the basics of complex numbers. The student will learn what a complex number is, how to represent these numbers in both the Cartesian and polar coordinate systems, how to convert between these systems, and how to perform basic arithmetic operations using complex numbers.

Chapter 3, the final math review chapter, introduces the student to some basic concepts in linear algebra. The chapter begins with the introduction of vectors and some simple vector arithmetic. The focus is then expanded to include matrices. Basic matrix arithmetic, such as addition, subtraction, and multiplication, is covered. Other basic matrix operations are also introduced. These include the concepts of matrix transpose, minors, cofactors, determinants, and inverses. The chapter concludes with an application of the matrix inverse. The use of the matrix inverse to obtain the solution of systems of linear equations is covered and related to the practical engineering application of circuit analysis.

Once the basics in mathematics have been established, the student may move on to the other four units of the text. The second unit consists of three chapters that are designed to provide the student with a better understanding of basic concepts in the area of electrical engineering.

Chapter 4 focuses on providing the student with the basics of electricity. In addition, the electrical components that are used to build electronic devices are examined.

Chapter 5 provides the student with an overview of electrical power and the power industry. The student is exposed to topics including the generation and supply of electrical power to the end user.

Chapter 6 introduces the student to the concepts involved in basic circuit analysis. They are first presented with the basics of resistors in series and parallel and taught how to find the equivalent resistances of these configurations. They then learn about voltage and current dividers and Kirchoff's voltage and current laws. Finally, the student is presented with two approaches to more complicated circuit analysis. The basics of both mesh and nodal analysis are presented.

The third unit of the text is designed to provide the student with a background in important computer engineering concepts. Its two chapters provide the student with basic concepts in the areas of digital logic and computer organization.

Chapter 7 presents the basic concepts of digital logic. The student is introduced to the basic logic gates, truth tables and Boolean expressions. They are taught how to analyze a logic circuit to create truth tables and how to build a logic circuit that satisfies a given set of conditions. Finally, students are taught the basics of logic simplification and introduced to the practical considerations involved in the construction of a logic circuit from physical components.

Chapter 8 continues with the student's introduction to computer engineering topics by describing the organization of a typical computer. The various functional components that make up a computer are described and the student is introduced to the way in which these components work together to perform the computer's basic functions.

The fourth unit covers the areas of digital signal processing, image processing, and communication. The four chapters in this unit provide the student with an overview of DSP, sound and image processing, communication theory, and information coding.

Chapter 9 is designed to provide the student with a basic introduction to the area of digital signal processing. The student is presented with an introduction to signals and systems. In addition, some of the basic operations involved in basic signal processing are introduced on a limited scale.

Chapter 10 provides an application of digital signal processing to the area of sound. The concept of audio signals is introduced, and several applications of digital signal processing within this area are presented.

Chapter 11 continues the examination of digital signal processing applications by looking at two-dimensional signals or images. The basics of photography and human vision are presented to show the importance of analog signals. Digital equivalents are given and the concepts of filtering, edge detection, and noise removal are explored.

Chapter 12 deals with the concepts involved in communication. The five parts of a basic communication system are examined. In addition, this chapter also explores the concept of encoding information for ease of transmission and the detection or correction of errors. Several coding techniques are presented.

The fifth and final unit provides several laboratory tutorials (modules), exercises, and design problems written to provide students with an introduction to several concepts and tools they will be utilizing throughout their education in engineering.

Chapter 13 is a self contained Module that introduces students to the MATLAB software package. Topics covered include MATLAB's basic commands, functions, plotting, and MATLAB M-Files

Chapter 14 introduces students to the CAD software package Multisim. Several topics are included such as voltage, current, power, and resistance measurements. This material can be used as a precursor to having the students make real measurements in the laboratory.

Chapter 15 introduces students to serial communications and the RS232 standard utilizing a personal computer and the Windows communication software HyperTerminal.

Chapter 16 covers the topic of operational amplifiers. It provides students with enough information to use op amps as building blocks in simple circuits. Several basic op amp circuits are discussed including the inverting, non-inverting, current to voltage, and summing amplifier configurations.

Chapter 17 introduces basic diodes. It introduces the general purpose silicon diode, light emitting diode (LED), and photodiode. In section 2 (optional) the characterization of semiconductor materials and a more detailed explanation of the pn junction are provided.

Chapter 18 introduces the concepts needed to understand the basic behavior of light and how fiber optic cable acts as a waveguide. The topics of light reflection, refraction, and total internal reflection are discussed.

Chapter 19 introduces students to the concept of PCB (Printed Circuit Board) design utilizing the software package ExpressPCB, A package available for free download at the ExpressPCB website. The software is introduced by having the student enter a simple predefined board design.

Finally, Chapter 20 contains several exercises, and an example design exercise is also presented. The exercises have been designed to provide students with practice applying the material presented in Chapters 13 through 19.

The goal of each of the chapters in this text is not to give the student an in-depth understanding of the topics that they cover, but rather to expose the student to the basic ideas and concepts at a level that their background allows.

Usage

The organization of this text allows instructors to present the material in a variety of different ways. Each of the chapters is self-contained and can stand alone should an instructor desire to present it in such a way. The recommended approach to this text is to present the first three chapters (Unit 1) first. After the students have the required math background, the remaining chapters can be approached in virtually any order.

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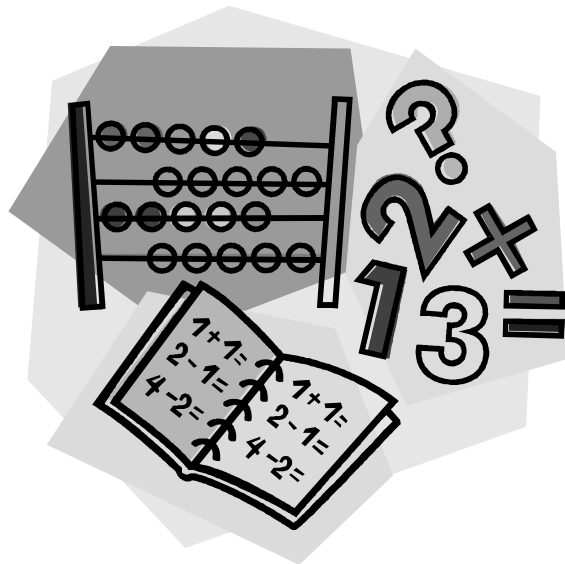
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UNIT 1



Basic Mathematics

Chapter 1. Numeric Bases

Chapter Outline

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1.1 Introduction to Numeric Bases

No matter what your occupation, you will encounter and work with numbers on a regular basis. Whether you are making change at a cash register, balancing your checkbook, or calculating power consumption of an electrical circuit, you must have a basic understanding of how numbers work. Take for example, the simple task of counting from 0 to 100. While this is a trivial task, you must know something about numbers to accomplish it. You begin counting using a single placeholder, the “ones” place. You know that when you reach the largest digit that can fit in this place (in this case 9) you must add an additional placeholder to continue. Thus, to proceed beyond 9, you add the “tens” place. One is added to this new place, and the original “ones” place is reset to zero. By doing so, you can proceed from 9 to 10. Likewise, each time you reach a 9 in the “ones” place, the next number is determined by incrementing the “tens” place and resetting the “ones” place. Following this procedure, you can count all the way up to 99. At this point, an additional placeholder, the “hundreds” place is used. By extending this procedure, it is possible to count to numbers far larger than 100.

While this example may seem obvious, it illustrates an important point about numbers. As you saw, there was a particular number that, to be properly represented required the addition of an additional placeholder. In this example we found that each time a placeholder reached the

number 10, we had to add a new placeholder to our number in order to represent it. When the “ones” place reached 10 we had to add the “tens” place, and when the “tens” place reached 10, we had to add the “hundreds” place. The number that, once reached, requires this addition of a new placeholder is called the radix. So, from our example, we find that the numbers we are used to working with have a radix of 10. Knowing the radix gives us important information about the number system that we are using, thus, the radix number is often referred to as the base of the number system. In other words, we typically work with numbers using a radix of 10, so we can say we use a base 10 number system. This base 10 system is often referred to as the decimal number system (from the Latin word “Decima” which means a tenth).

The decimal number system is not the only one used, however. Many applications, especially in electrical engineering and other technical fields, require the use of number systems based on a different radix. One good example is the binary, or base 2, number system. It is commonly used in the areas of digital logic and computers. These areas deal with signals that can have only two possible states, on or off. Thus a number system with a radix of two is perfectly suited for describing events in these systems.

Before we look too closely at the use of other bases, let’s get a better understanding of the mechanics of numbers in the decimal system. In order to see exactly how decimal numbers work, let’s look at an example of a typical decimal number. In this case, let’s use the decimal number $(1279.875)_{10}$. You may have noticed that this number is not written quite the way you might expect. The parentheses and subscript 10 is part of a notation method that is often used when dealing with numbers of different bases. It helps us avoid confusion by keeping track of what base each number is in. If we did not specify the base each number is in, it would be easy to get confused, and that could lead to unwanted errors. Now that we are comfortable with this notation, let’s look at what this decimal number is really telling us. Figure 1.1, shown below, breaks the number down into its respective places.

	1	2	7	9	.	8	7	5
	10^3	10^2	10^1	10^0		10^{-1}	10^{-2}	10^{-3}
Thousands	1000						0.001	Thousandths
Hundreds	100						0.01	Hundredths
Tens	10						0.1	Tenths
Ones	1							

Figure 1.1. An example of a typical decimal (Base 10) number.

As you can see, by breaking the number down, we find that each of the digits gives us important information. The numeric value in each of the placeholders indicates the degree to which its respective power of the base is represented in the total number. In this example, it is easy to see that there are 1×10^3 or 1 “thousand,” 2×10^2 or 2 “hundreds,” 7×10^1 or 7 “tens,” 9×10^0 or 9 “ones,” 8×10^{-1} or 8 “tenths,” 7×10^{-2} or 7 “hundredths,” and 5×10^{-3} or 5 “thousandths” in the number. As a result the final number can be thought of as $(1 \times 10^3) + (2 \times 10^2) + (7 \times 10^1) + (9 \times 10^0) + (8 \times 10^{-1}) + (7 \times 10^{-2}) + (5 \times 10^{-3})$ or $1000 + 200 + 70 + 9 + 0.8 + 0.07 + 0.005 = 1279.875$. While this may seem obvious when working with decimal numbers, the same concept can be used to work with numbers of any base. To better understand this, let’s first look at how this works for an arbitrary base b . We will then examine numbers in some of the more commonly used bases.

1.2 Working With Arbitrary Bases

While working with numbers in a specific base is useful, it is far more beneficial to be able to work with numbers in any base. Since you never know when you may encounter numbers in an unfamiliar base, it is important to have general tools that can be adapted to work with any base you choose. In this section, we will explore the basics of bases. We will learn how counting works in different bases, and we will learn how to convert numbers between an arbitrary base and decimal.

1.2.1 Counting in an Arbitrary Base

Once we have determined what base we are working with, let's call it b , we automatically know two important things about our number. First, we know that each digit or placeholder in the number is going to represent a particular power of the base, and second, there are exactly b values that can be used in each placeholder. These values will range from 0 to $(b - 1)$. As a result, when we count in a base b number system, each placeholder will begin at 0 and increment up to $b - 1$. Once a placeholder has reached $b - 1$, a new placeholder must be added to count beyond that point.

To help clarify this concept, let's look at an example.

Example 1.1

Problem: Count to $(20)_{10}$ in base 5

Solution: Since we have been given a base to work in, we can assign a value to our arbitrary base b . In this case let $b = 5$. Since we now know that b is equal to 5, we know that each digit of any number represented in this base gives us an indication of the degree to which each power of five is represented in the number as a whole. We also know that each digit of the number will range from 0 to $(5 - 1)$ or 4. So, let's count to $(20)_{10}$ in base 5.

Decimal	Base 5
0	0
1	1
2	2
3	3
4	4

As you can see we have reached the value of $b - 1$, which in this case is 4. Since our radix is 5, we know that when a placeholder reaches 5 a new placeholder must be added to represent the number. Let's add a placeholder and continue.

Decimal	Base 5
5	10
6	11
7	12
8	13
9	14
10	20

At this point, we have reached $(20)_5$. However, this does not mean we have finished counting. If we look at the table, we will see that $(20)_5$ is equivalent to only $(10)_{10}$. We are only half-way there. Let's count the rest of the way now.

Decimal	Base 5
11	21
12	22
13	23
14	24
15	30
16	31
17	32
18	33
19	34
20	40

As you can see, we have now reached $(20)_{10}$ in the base five number system. From the chart we see that $(40)_5$ is equal to $(20)_{10}$.

This example gives us a good idea of how counting in a base other than decimal works. At this point you may ask, is there any way to tell what a number in some other base b will equal in decimal without building a chart and counting? The answer is yes.

1.2.2 Converting an Arbitrary Base to Decimal

The process used to determine the decimal equivalent of a number expressed in a different base b is actually very simple. The key is to remember the function of each of the placeholders in the number. Each placeholder determines to what degree a certain power of the base is represented in the final number. In addition, each placeholder to the left of the radix point (note that this is what a decimal point is called in number systems that are not base 10) represents an increasing positive power of the base, and that each placeholder to the right of the radix point represents an increasing negative power of the base. Let's take a look at what this would look like for a general base b .

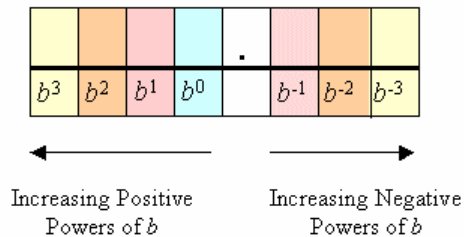


Figure 1.2. An illustration of the relationship between each placeholder and its respective power of the base.

Since each placeholder represents a specific power of the base, we can obtain the decimal value represented by a given placeholder by simply multiplying the value in that placeholder by the value obtained by raising the base to the appropriate power. If we calculate this value for each placeholder and sum the results, we will arrive at the decimal equivalent of the number represented in base b . To illustrate this, let's look at base 5 once again. Figure 1.3 displays the powers associated with each placeholder in the base 5 system.

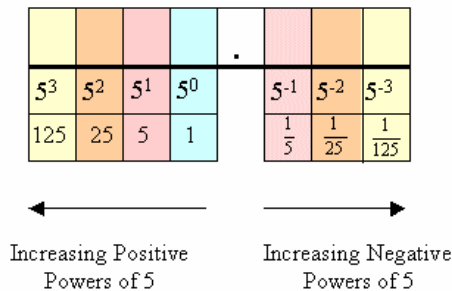


Figure 1.3 A representation of the powers of 5 associated with the placeholders in a base 5 number.

Continuing with our base 5 example, let's find the equivalent decimal number for $(13412.213)_5$.

Example 1.2

Problem: Find the equivalent decimal number for $(13412.213)_5$.

Solution: Our first step in converting this number to its decimal equivalent is to multiply the value in each placeholder by its respective power of the base, in this case 5. The following table illustrates this process.

Placeholder Value	Associated Power of the Base	Resulting Product
1	$5^4=625$	$1 \times 625 = 625$
3	$5^3=125$	$3 \times 125 = 375$
4	$5^2=25$	$4 \times 25 = 100$
1	$5^1=5$	$1 \times 5 = 5$
2	$5^0=1$	$2 \times 1 = 2$
.	Radix Pt.	
2	$5^{-1}=0.2$	$2 \times 0.2 = 0.4$
1	$5^{-2}=0.04$	$1 \times 0.04 = 0.04$
3	$5^{-3}=0.008$	$3 \times 0.008 = 0.024$

Now that we know the decimal value associated with each placeholder, all we have to do is add these values together to arrive at our answer.

$$\begin{array}{r}
 625.000 \\
 375.000 \\
 100.000 \\
 5.000 \\
 2.000 \\
 0.400 \\
 0.040 \\
 + 0.024 \\
 \hline
 1107.464
 \end{array}$$

So, now we know that $(13412.213)_5$ is equivalent to $(1107.464)_{10}$.

In the preceding example, we found the decimal equivalent of a base 5 number. The method we used in this example will work to convert a number in any base to its decimal equivalent. Simply multiply the value in each placeholder by its respective power of the base and add the results.

MATLAB: MATLAB has a function that allows for the conversion of an arbitrary base to its decimal equivalent number. This function has the following usage syntax.

BASE2DEC(S,B)

Where S is a string containing the number in base B, and B is the base that the number is in. Keep in mind that this function only converts positive integer values. It will not work on negative numbers or numbers containing fractional parts.

Example: Base2Dec('13412',5) will return 1107 which is the decimal equivalent of the specified base 5 number.

1.2.3 Converting Decimal to an Arbitrary Base

Now that we know how to find the decimal equivalent of a number given in another base, our next goal is to learn how to work in the other direction. We want to be able to represent a given decimal number in another base. The procedure required to perform this conversion is slightly more complicated than the conversion to decimal, but it is still not too difficult. Unlike the process of converting a given base to decimal, the reverse process requires separate processing for the whole and fractional parts of the number.

Let's look at the whole number part first. The easiest way to determine the representation of the whole number portion of a decimal number in a different base is by successively dividing the decimal number by the radix of the new base and keeping track of the remainder. This sounds difficult at first, but is actually quite easy. If we wish to perform the conversion of a whole number from decimal to base b , we must divide our number by b . The remainder of this division will be the rightmost digit of our base b number. Next, we divide the quotient from the previous division by b . The remainder from this division is the next digit in our base b number. This division process is repeated until the quotient is zero. At this point we have successfully converted our decimal number to the new base. Figure 1.4 illustrates this process.

$$\begin{array}{r}
 0 \\
 \hline
 b \overline{) Q_3} \\
 \hline
 b \overline{) Q_2} \\
 \hline
 b \overline{) Q_1} \\
 \hline
 b \overline{) Decimal}
 \end{array}
 \begin{array}{l}
 R_4 \\
 R_3 \\
 R_2 \\
 R_1
 \end{array}$$

$$(\text{Decimal})_{10} \longrightarrow (R_4 R_3 R_2 R_1)_b$$

Figure 1.4. Division method for base conversion.

Having completed the conversion of the whole number portion of the decimal number, we can now focus on the fractional portion. While the whole number portion was converted using division, the fractional portion of the number is converted using multiplication. In this case, the fractional part of the number is multiplied by the radix of the new base. After the multiplication we record the whole number portion of the result. Then, if the fractional part of the number is different from any of the previously encountered fractional parts, we continue to multiply and track the whole number portion. It should be noted that the multiplication at each stage is applied to only the fractional part of the number. Any whole number portion from previous multiplications is ignored. The process comes to an end when either, the fractional portion of the number becomes zero or, a duplicate fractional portion is encountered. If the fractional portion of the number becomes zero, the conversion to the new base was successful and the resulting number can be represented with a finite number of placeholders. If a duplicate fractional portion is encountered, the representation in the new base requires a repeating number (Infinite Placeholders). In this case, all the numbers between the first and second occurrence of the duplicate fractional part will repeat. Figure 1.5 illustrates the procedure.

$$\begin{array}{l}
 0.dddd_1 \\
 \times \quad b \\
 \hline
 w_1.dddd_2 \\
 \times \quad b \\
 \hline
 w_2.dddd_3 \\
 \times \quad b \\
 \hline
 w_3.0000
 \end{array}$$

$$(0.dddd)_{10} \longrightarrow (0.w_1 w_2 w_3)_b$$

Figure 1.5. An illustration of the conversion of fractional numbers to a new base.

The best way to truly understand the conversion from decimal to another base is by actually looking at an example. First, let's look at the number $(1107.464)_{10}$. This is the number that we arrived at after our conversion from base 5 to decimal in the previous example. If we use the