Initiation and Control of Gait from First Principles:

A Mathematically Animated Model of the Foot

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ABSTRACT

The initiation and control of gait from first principles: a mathematically animated model of the foot, a doctoral thesis by Craig Nevin examines the anatomical locations of the dynamic pressures that create the first five footprints when a standing person starts to walk. It is hypothesized that the primary activity starts with the dorsiflexion or lifting of the great toe. The metatarsophalangeal region of the forefoot where the first action is believed to begin was studied from three directions.

- Viewed side-on, the great toe free-body is found from a detailed post hoc analysis of previous kinematic data obtained from cadavers to operate as a cam. The cam model also follows closely from Aristotle's ancient description of the hinged instrument of animate motion.
- In cross-section, the first metatarsal *torsion strength* was estimated in 13 human, 1 gorilla, 3 chimpanzee 1 orangutan and 1 baboon set of dry-bone specimens of the hands and feet. The first metatarsal bone alone contributes 43% of the total strength of all the metatarsal bones. A result unique amongst the hominids and apes studied.
- The dynamic components and principle axes of the footprints of 54 humans (32 years ±11, 32 male, 22 female) were studied whilst standing on a 0.5m pressure plate, and then immediately when walking over a 2m plate (4 sensors per cm^2 sampled at 100hz). Two footprints were obtained during the initial stance posture, and the first three footprints of the initial walk.

Three new *principles of animate motion* were deduced from the divergent results obtained from complete and dissected cadavers: The *metatarsal cam* (from the sagittal side view) the *ground reaction torque* (from the frontal coronal view) and the *amputation artifact.* The philosophy of experimenting on inanimate cadavers rather than living subjects was intensively researched.

Instead of assuming that gait is a uniform or regular motion as is usual, the foot was analyzed rather as if it was a beam attached the ground. Engineering equations were used to determine the flexural properties of the foot every 0.01 seconds, including the principle axes, radius of gyration and the local shear stresses on the sensors spaced 5-7mm apart. A sequence of these impressions creates a mathematically animated model of the footprint.

The local force under the foot was normalized against both the total force and contact duration. The forces under the foot were each divided between 10 anatomical regions using individual masks for each foot strike. Producing a 54-subject database from which the normal behavior of the foot could be quantified. The group showed a surprisingly low right foot dominance of only 54%.

The combination of the radius of gyration and impulse in particular produces a succinct but powerful summary of the footprint during dynamic activity. The initial angle and magnitudes of the loads that are applied and removed demonstrates that the body first rocks onto the heels after the instruction to walk is given. The feet simultaneously invert and their arches rise off the ground as anticipated.

The principle axes were then animated in a mathematical four-dimensional model. The horizontal radius of gyration is on average 5 cm during heel strike, but increases to 20 cm as the forefoot comes into contact with the ground, finally rising to 25 cm at toe-off. Significantly the applied load during the fore-foot loading phase is more widely distributed than the load being removed. A new and unanticipated result that is believed to be a special characteristic of the animate foot.

The standard deviation of the force under the great toe is the first mechanical parameter to converge in the 54 subjects. Conclusively verifying the hypothesis that the great toe both initiates and controls gait.

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GLOSSARY OF ABBREVIATIONS

$\{\ \}_1$	first ray of toe or finger, subscript identifier
$\{\ \}_2$	second ray of toe or finger, subscript identifier
$\{\ \}_3$	third ray of toe or finger, subscript identifier
$\{\ \}_4$	fourth ray of toe or finger, subscript identifier
$\{\ \}_5$	fifth ray of toe or finger, subscript identifier
1	metatarsophalangeal joint in cadaver preparation
2	junction between metatarsal base and foot, the cam follower at metatarso-cuneiform joint
3	junction between cadaver torso and dissection room bench **B**
4	junction between compliant element **D** and bench **B'**
5	junction between compliant support element **D** and the drawing board **A**
6	G-clamp between metatarsal and cam drawing board
7	sliding junction between drawing board and projected metatarsal and phalanx positions
8	clamp or tie between amputated specimen and experimental apparatus support frame **E'**
2D	two dimensional (x,y)
3D	three dimensional (x,y,z)
4D	any four dimensional system of coordinates; e.g. relativistic [3L+1T], Cartesian 4i *res cogitans*, *res extensa* second moment of area framework, tensegrity tetrahedron
A	frame where kinematic data is gathered during KPT experiment
$\mathbf{A}_1$	sagittal plane or side-view of kinematic frame **A**, the drawing board plane used for the cam
$\mathbf{A}_2$	transverse horizontal plane, view of kinematic frame **A**
$\mathbf{A}_3$	coronal or frontal view of kinematic frame **A**; cross section of forefoot
$\mathbf{a}_{12}$	anterior-posterior axis of kinematic frame **A** at intersection of planes $\mathbf{A}_1$ and $\mathbf{A}_2$
$\mathbf{a}_{13}$	superior-inferior axis of kinematic frame **A** at intersection of planes $\mathbf{A}_1$ and $\mathbf{A}_3$
$\mathbf{a}_{23}$	medial-lateral axis of kinematic frame **A** at intersection of planes $\mathbf{A}_2$ and $\mathbf{A}_3$
A	area
Å	area enclosed by a cross section of the outermost perimeter rim of bone shaft annulus
B	bench supporting cadaver
B'	bench supporting cadaver part, differing in height from bench **B** by height **h**.
BAF	body action force
C	cadaver torso proximal part
CoF	center of force
CoM	center of mass
CoP	center of pressure
D	compliant element of undetermined stiffness
d	distance variable
E	external experimental apparatus applying force
E'	external experimental apparatus applying force against own resistance
EHB	extensor hallucis brevis muscle
EHL	extensor hallucis brevis muscle
FDL	flexor digitorum longus muscle

F	foot part of cadaver
FDL	flexor digitorum longus muscle
FHB	flexor hallucis brevis muscle
FHL	flexor hallucis longus muscle
G	ground or gravity reference frame
$\mathbf{G}_1$	vertical side-view, gravity/ground reference
$\mathbf{G}_2$	horizontal plane, gravity/ground reference
$\mathbf{G}_3$	frontal cross-section view, gravity/ground reference
$\mathbf{g}_{12}$	horizontal fore-aft axis at intersection of planes $\mathbf{G}_1$ and $\mathbf{G}_2$
$\mathbf{g}_{13}$	vertical axis at intersection of planes $\mathbf{G}_1$ and $\mathbf{G}_3$
$\mathbf{g}_{23}$	horizontal transverse axis at intersection of planes $\mathbf{G}_2$ and $\mathbf{G}_3$
GRF	ground reaction force
GRT	ground reaction torque; ground reaction twist
HL	lateral half of heel footprint
HM	medial half of heel footprint
h	vertical distance between supported body **C** on bench **B** and non-compliant bench **B'** on ultimate ground support **G**; Galilean pedestal height
I	second moment of area
i	imaginary mathematical unit; $\sqrt{-1}$
J	polar second moment of area
KPT	kinematic projected transformation experimental method
L	isolated laboratory reference frame
[L]	length dimension
[M]	mass dimension
M	frame clamped to first metatarsal bone
$\mathbf{M}_1$	sagittal plane or side-view of metatarsal frame **M**
$\mathbf{M}_2$	transverse, approximately horizontal plane of metatarsal frame **M**
$\mathbf{M}_3$	coronal cross-section or frontal view of metatarsal frame **M**
$\mathbf{m}_{12}$	long axis of metatarsal bone at intersection of planes $\mathbf{M}_1$ and $\mathbf{M}_2$
$\mathbf{m}_{13}$	superior-inferior metatarsal axis at intersection of planes $\mathbf{M}_1$ and $\mathbf{M}_3$
$\mathbf{m}_{23}$	medial lateral anatomical axis at intersection of planes $\mathbf{M}_2$ and $\mathbf{M}_3$
M1-L	first metatarsal head region of left footprint
M1-R	first metatarsal head region of right footprint
M2-L	second metatarsal head region left of footprint
M2-R	second metatarsal head region right of footprint
M3-L	third metatarsal head region of left footprint
M3-R	third metatarsal head region of right footprint
M4-L	fourth metatarsal head region of left footprint
M4-R	fourth metatarsal head region of right footprint

M5-L	fifth metatarsal head region of left footprint
M5-R	fifth metatarsal head region of right footprint
MC	metacarpal bone in hand
MF-L	mid-foot region of left footprint
MF-R	mid-foot region of right footprint
MT	metatarsal bone in foot
MT-1	first metatarsal bone
MTP	metatarsophalangeal joint
MTP-1	first metatarsophalangeal joint
P	frame clamped to first proximal phalangeal bone
$\mathbf{P}_1$	sagittal plane or side-view of proximal phalangeal frame **P**
$\mathbf{P}_2$	transverse, approximately horizontal plane of proximal phalangeal frame **P**
$\mathbf{P}_3$	coronal cross-section or frontal view of proximal phalangeal frame **P**
$\mathbf{p}_{12}$	long axis of proximal phalanx at intersection of planes $\mathbf{P}_1$ and $\mathbf{P}_2$
$\mathbf{p}_{13}$	superior-inferior axis of proximal phalanx at intersection of planes $\mathbf{P}_1$ and $\mathbf{P}_3$
$\mathbf{p}_{23}$	medial lateral anatomical axis of the phalanx at intersection of planes $\mathbf{P}_2$ and $\mathbf{P}_3$
$\mathbf{P}_i$	universal joint clamped to phalanx to enable phalanx frame **P** to coincide with frame **A**
PL	peroneus longus muscle
q	shear force
R	outer radius
r	generic radius, inner radius
RoG	radius of gyration of footprint in horizontal ground plane
s	distance
T	tensile soft tissues in foot
T	torque
t	thickness; time
τ	shear stress
[T]	time dimension
T1-L	great toe phalanx region of left footprint
T1-R	great toe phalanx region of right footprint
T2-5-L	lesser toe phalanx regions of left footprint
T2-5-R	lesser toe phalanx regions of right footprint
TA	tibialis anterior muscle
TP	tibialis posterior muscle
X	x coordinate of center of pressure
x	cartesian *res extensa* primary horizontal coordinate
Y	y coordinate of center of pressure
y	cartesian *res extensa* secondary horizontal coordinate
z	cartesian *res extensa* vertical coordinate

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1.
INTRODUCTION

> *"It seems reasonable to begin the description of walking with a discussion of the translation of the body as a whole through space." — Inman, Ralston and Todd*[1]

1.1 BACKGROUND

Starting to walk from a standing posture is an effortless task. But the scientific problem of how we start to walk is much more complex, not least because it is difficult to explain how disembodied thoughts exert any control whatsoever over the motion of our physical bodies.[2] The basic problem is not a new one. Zeno of Elea (470 BC) for example pointed out a famous paradox[3] that arises when one attempts to study the motion of an elite athlete for example by dividing it into finite intervals.

The modern gait analysis method arguably began though with Braune and Fischer (1896-1904), whom were the first to combine the then new technology of cinematic animation[4] with Newton's mathematical principles. This cinematic animation method however relies precisely on the type of finite time-interval data that Zeno found so paradoxical. What's more modern cinematic *animation* is barely recognizable as the *anima* that the ancient believed was the *first cause* of all movement.

Aristotle (384-322 BC) wrote several books on the *Anima*, although he is perhaps better known as the author of the original *Physics*.[5] The influence of the *Physics* lasted almost two thousand years until Johannes Kepler (1621) choose to describe the universal *anima,* the principle that was assumed to move the heavens, as a *physical force*.[6] Soon thereafter Galileo Galilei (1638) launched the new mechanical sciences,[7] forging a reputation as an experimental philosopher at Aristotle's expense. French philosopher and mathematician René Descartes (1644) also rejected the old interpretations of the Aristotelian Schools and sought to explain human movement in terms of mechanics and physiological reflex. But it was ultimately Isaac Newton (1687) who replaced the theology of the heavens animated by cherubs and angels with the mathematical principles of mechanical physics.

Ironically, Galileo and Newton's hypothesis of *uniform motion,* which superceded the physics of Aristotle's *unmoved mover*, has itself long been eclipsed by the modern physics of quantum mechanics and relativity theory. But in biomechanical schools where one deals with physics on a human scale, these newer and older theories are known only through occasional anecdotes. It is very much the scientific view that "Newton rules biology"[8] despite the fact that Newton unlike Aristotle wrote very little on the movement of animals, and even that which he specifically did write has been almost entirely disregarded in favour of his mathematical principles of motion.

Unlike gait itself, which has been studied extensively through cameras, the initiation of gait phase has mostly been studied experimentally using electrodes on the muscles, and a force plate beneath the feet to measure the ground reaction force. These methods presuppose that gait is under *active neuromuscular control* and best described in terms of propulsive Newtonian forces. So it is sensible to first see what can be inferred from these data.

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[1] *Human Walking*, 1st edition (1981).

[2] Mario Bunge (1980) writes "The mind-body problem is a notoriously hard nut to crack—surely even more so than the problem of matter."

[3] The 'Achilles paradox'.

[4] Muybridge (1887).

[5] Aristotle's texts have historically been divided between the *Physics* and the *Metaphysics*. Not as I propose *Physics* and the *Anima*.

[6] Jammer (1957), p.90.

[7] Galilei (1638) *Dialogues and mathematical demonstrations concerning two new sciences pertaining to mechanics and local motions*. See Ascenzi, (1993).

[8] Pennycuick (1992).

1.1.1 PRINCIPLE PREMISE

Newton's first law requires that a force be impressed to move a body from a state of rest. Impressed forces arise specifically according to Newton from pressure, percussion and centripetal forces such as gravity.[9] Newton's third law of motion suggests that for every action there is an equal and opposite reaction directed to contrary parts. Thus, if the human body initiates action, like starting to walk, there must be a ground reaction effect exactly in the opposite direction. A simple plot of the progression of the *ground reaction force* [Fig. 1-1] indicates that the causal *body action force*, which is presumed to be the motivation behind the movement, is directly aligned with the first metatarsophalangeal joint (MTP-1) of the great toe of stance foot.

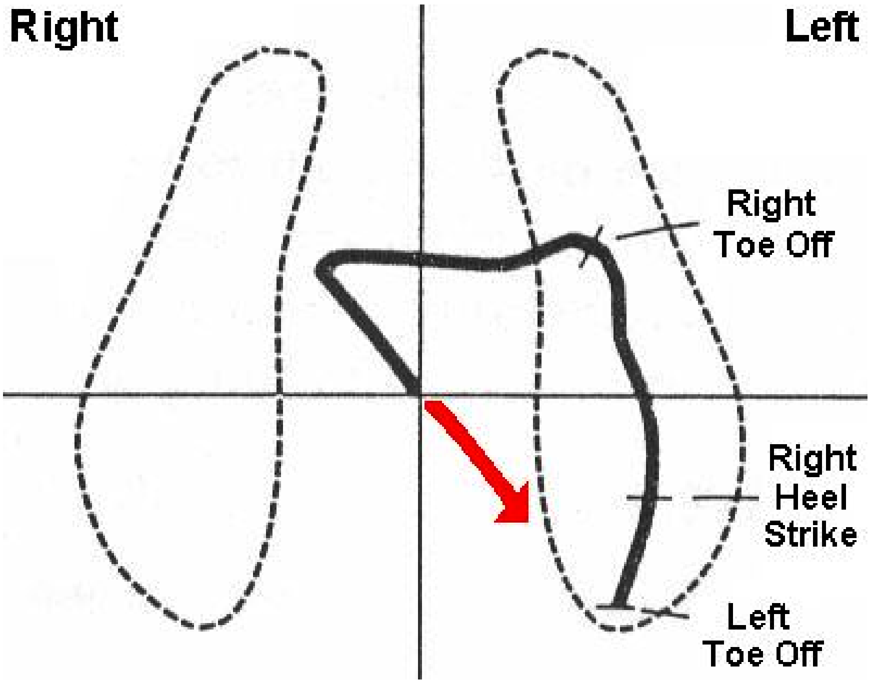

Fig. 1-1. The hypothetical body action force (arrow) that acts through the great toe of the initial stance foot. The arrow in the left forefoot quadrant is superimposed here on a center of force diagram adapted from Mann et al. (1979).

The *principle premise* of the thesis is therefore that the anatomical, physiological, mathematical and biomechanical mechanisms needed to animate the body via a voluntary human bipedal body action force can be determined directly from an analysis of the motion of the great toe.

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[9] Newton (1726). *Principia*, Definitions 3-4.

1.2 ORGANISATION OF THESIS

A thesis has several forms. An academic thesis is simply a written dissertation advancing an original point of view as a result of research leading to an advance in knowledge.[10] From this definition three basic requirements emerge:

- The need for a *new* and *original point of view*.
- A description of how this *original point of view* is based on the *results of research*.
- A demonstration of how this *new* and *original point of view* leads to an *advance in knowledge*.

1.2.1 ADVANCE IN KNOWLEDGE

Given the academic requirement for advancing knowledge, the dialectic thesis format of Hegel is preferred. Whereby a proposition (thesis) is transformed into its opposite (antithesis) and preserved and fulfilled by it, the combination of the two being resolved in a higher form of truth (synthesis); the ultimate synthesis being that of the mind or thought.[11] Another good reason for preferring the dialectic format is that philosophers such as Hamblin (1963) recognize that stopping and starting is not really an empirical problem at all and hence unlikely to be resolved by a simple discussion of empirical methods and materials.[12]

The two types of thesis, dialectic and academic, also start with different philosophical premises:

- *Academic* premises are based entirely on original observations of experimental phenomena.
- *Dialectic* premises follow directly from the perceived fallacies and deficiencies of the philosophical foundations of the opposing views.

Besides these traditional tests of knowledge, reference is also made to Descartes' *Principles of philosophy*, particularly those parts *On human knowledge,* and Oldroyd's (1986) thesis of an *Arch of knowledge,* which is based in part on Newton (1730) Question 31 in the *Opticks*.[13] The arch of knowledge model allows a succinct comparison between the higher structures of intellect required for Hegel's synthesis and the anatomical concept of a bony arch of the foot and its plantar supporting structures that provide the subject material for this thesis.

1.2.2 ORIGINAL POINT OF VIEW

The basis of a thesis is an original hypothesis. But the word *original* has at least four meanings:

- **Novelty**. A unique or innovative method or observation.
- **Precedence**. The first of its kind, attributable to a specific point in time, a historic date or event.
- **Mathematical** origin. A dimensionless mathematical point, with no property but position.
- **Anatomical** origin. Start of a nerve or muscle; the proximal or immobile end, as opposed to its distal mobile *insertion*.

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[10] **thesis**; Reader's Digest Universal Dictionary.

[11] **dialectic**; Reader's Digest Universal Dictionary.

[12] An empirical scientific thesis traditionally draws *conclusions* from a *discussion* of *results* after having applied certain *methods* to specific *materials* following from an *hypothesis* that is established as novel by an exhaustive *review* of the literature.

[13] Oldroyd (1986, pp.78-84).

Most important of all is the mathematical point—the *original dimensionless point* from which the *original point of view* of a scientific thesis ultimately depends. This point, which has no property except position, nevertheless has to be linked with appropriate anatomical and historical origins.

The *novel anatomical origin* is the distal proximal phalanx of the great toe. In a historical sense, this origin follows logically from Aristotle's physical solution of the mind-body problem in his third book *On the Anima* where he described the origin of motion arising in an animate hinge joint. But this choice of historical origin in the *Anima* is unique because Aristotle in the *Progression of animals*[14] preferred to locate the origin *proximally* near the heart, not in the *distal* segment of the toe. Vainly contriving to establish an honorable hierarchical 'proper place' for the origin of human motion.

Consequently in anatomical schools, the *origin* of a nerve or muscle is fixed in the proximal segment, with the assumption that it moves the *insertion* at the distal segment. But in reality it is often the distal segment that is fixed on the ground during gait and the proximal segment that moves most. By shifting Aristotle's original *origin of movement* to the distal portion of the great toe at rest, the thesis begins the process of reversing the ancient hierarchy of the theory of human movement control in an original anatomical, mathematical and historical sense.

1.2.3 RESULTS OF RESEARCH

The *novel method* that lays the phenomenal foundations for the present thesis, comes from a prior dissertation of mine.[15] I shall refer to this method as kinematic projected transformation (KPT). It is novel because it provides an unprecedented description of the *relative rotations* of the inanimate cadaver great toe. The KPT method basically provides experimental support for Aristotle's theory of the *unmoved mover* applied to an *animate hinge*.

Another novel aspect arises specifically when Aristotle's arbitrarily elevated 'proper place' for the soul is discarded and replaced by an ostensibly less honorable *osseo-aponeurotic* model of gait initiation and control centered in the great toe. This philosophy originates in my reading of Hick's (1954) who provided a new and original description of the functional anatomy of the great toe in the dead and paralytic where active neuromuscular control was entirely absent.

The thesis in essence combines the results of this previous research in the mechanics of the inanimate MTP-1 with Aristotle's description of a hinge joint, which is then mathematically re-animated using an extended set of Cartesian coordinates physically attached to the great toe.

1.3 THESIS OVERVIEW

Aristotle was at the forefront of the academic tradition to first review the views of others.[16] Einstein noted that new insights and theories develop gradually and should not be compared to knocking down an old barn and building a sky-scraper in its place.[17] Perhaps here a comparison can be made with a hiker walking along a wilderness trail through a jungle of data. The "views" are panoramas from existing paths. The veritable jungle of data can be viewed close up from the paths with all its snags, thorns and bugs; or overviewed as a distant vista from a clear supposedly well-established historical vantage point. The well-trodden paths constitute the received view, academic starting points that lead to specific view-points. The data may be said to be viewed or reviewed respectively depending

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[14] 706b.

[15] Nevin (1995). The design and cadaveric assessment of a new artificial first metatarsophalangeal joint replacement for the great toe. Masters dissertation.

[16] Zeno's paradox's are for example preserved through Aristotle's review.

[17] *The evolution of physics*, Einstein and Infeld (1971) p.152.

on one’s current or historical viewpoint and the direction of progression relative to these official route indicators or established traffic flow.

At specific junctions the historic trails naturally diverge. One deliberate or implicit choice of intellectual direction at a dichotomous junction leads to the view point of the thesis and the other to the opposite view point of the antithesis. In terms of the Hegelian synthesis these *view points* are just different vistas of the same mountain of experimental evidence. It is ultimately the combination or synthesis of these disparate views that increases to our knowledge by providing a more comprehensive map of the subject matter.

The function of the *review of the antithesis* in Chapter 2 is not to busy ourselves preventing weed encroachment by eliminating opposing views at the most popular neo-Newtonian viewpoint. But rather to open up new and often surprisingly old trails, all the time *re-viewing* or viewing afresh the data back to an historical *starting point* from whence one can synthesize a novel solution to the problem of initiating gait.[18]

1.3.1 VIEWS OF THESIS

The first observation is that gait is invariably viewed as a state of *uniform motion*, despite the fact that there is arguably no experimental evidence to suggest that a state of uniform motion is ever achieved. The very words walking and running for example imply motion. Hence it is exceedingly difficult to distinguish how the concept of walking or running for example differs from the motion itself.[19] Uniform motion has by definition no beginning or end.[20] Hence the common view that gait is a state of uniform motion, actually creates the problem under scrutiny, that of initiating gait.

The *uniform motion* postulate thus forms the natural antithesis to the thesis. Paradoxically, any true solution to the initiation of gait problem is compelled to undermine the postulate of uniform motion. This is not as simple as one might suppose, because the postulate of uniform motion otherwise known as the Galilean invariance, forms one of the main foundations of classical physics.[21]

One could argue the point dialectically for example, by taking an academic position contrary to Galileo or Newton’s fundamental premise of uniform motion.[22] But I forgo this opportunity; conceding instead that the premises of Galileo and Newton are as valid as just every academic presupposes. But then I go further to hypothesize that the neo-Newtonian school of researchers that has championed these theories, have failed to apply Newton's philosophy with *sufficient* rigor.

Rigorous application of Newton hypotheses of animal motion for example requires a consideration of *aether* and the *Power of the Will* and several other factors such as the mechanisms of *anatomical choice* that are touted in Question 31 of his *Opticks*. All of which are almost completely ignored by existing classical biomechanical models with Newtonian pretensions. Galileo for example, in his famous example of his invariance aboard a ship, concludes that one cannot distinguish between moving uniformly and standing still. Yet through the classic gait and posture divide, biomechanists routinely do. The primary criticism of the antithesis therefore is based on simple self-contradiction in the philosophical foundations in the biomechanics rather than technical imprecision in its application.

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[18] I am indebted to Prof. Laurie Adams for the insight that any robust claim to originality (novelty) is more than likely to be an unfortunate admission of an incomplete literature review.

[19] Many philosophers throughout history have used walking specifically as *an example* of a state of motion. Opposing it to a state of rest.

[20] An inference exploited by Zeno.

[21] The term ‘classical physics’ applies here to the sense used by Penrose (1989), i.e. to continuum mechanics as distinct from quantum mechanics.

[22] The traditional scholastic dialectic tradition, of arguing a thesis from the false premises of the opposing view, necessarily leads to a situation where Newton’s hypothesis of equal and opposite reactions for example can be right only if Aristotle’s hypothesis of an unmoved mover was wrong.