Cognitive Units, Concept Images, and Cognitive Collages:

An Examination of the Processes of Knowledge Construction

Mercedes A. McGowen

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Table of Contents

List of Tables

List of Figures

Acknowledgments

The moral is it hardly need by shown,
All those who try to go it sole alone,
Too proud to be beholden for relief,
Are absolutely sure to come to grief.
– Frost, *Moral*

The writing of this dissertation has been a challenging adventure—across seas, venturing forth to explore new vistas, making new discoveries, mathematical, visual, and musical, fulfilling long-held dreams, and at home—facing the herculean task of assembling the bits and pieces of new knowledge into the cognitive collage that is this thesis—assembled with the threads of intuition and analysis, shaped by the ambered heat of debates, and refreshed by quiet reflections down peaceful roads, alone and with colleagues and friends.

It has required a delicate juggling act—trying to balance my roles of student, observer, listener, and researcher, with those of wife, mother, teacher, friend and author. The debt of gratitude I owe to numerous friends, colleagues, and my family for their patience, support, humour and generosity is one I acknowledge but cannot hope to repay.

To my thesis advisor, Dr. David Tall—my sincere thanks and appreciation for a most remarkable four years of adventure and challenge and joy. His writings opened up the world of mathematics education research for me. His knowledge, expertise, humour, patience, and encouragement, standards of excellence, and scholarship set me on journeys, intellectual and historical, which I shall long remember and which I shall try to live up to. His love of music and drama introduced me to the music of the night. May the challenges and adventures long continue!

I wish to thank Dr. Eddie Gray for his advice, insights, and critiques which proved invaluable—and inspired me to think about ways to present the data of this dissertation with greater clarity, which, it turned out, was the key to making sense of it after all.

To my colleagues and, in particular, my textbook co-authors, Phil DeMarois and Darlene Whitkanack, my heart-felt gratitude for their friendship and shared knowledge. They have nourished me intellectually, encouraged me professionally, and sustained me in moments of doubt.

To Keith Schwingendorf—my thanks for introducing me to the writings of David Tall, in the rain at Allerton, and during the remaining days of the conference.

I owe much gratitude to the students who have participated in this research study. They gave generously of their time and of themselves. They were wonderful teachers and supportive partners in this research.

Finally, I thank my husband, Bill, my sons, Bill, Mike, Tom, and John, and my extended family, including Jennifer, Laura, Debbie, and Molly, for their love, patience, and support. This thesis would never have been completed without their generosity and friendship. They are my compass—as I went forth on my adventures, constructing new cognitive collages of thoughts, caught up in the plethora of details, deadlines, and tasks yet-to-be-done—they were always there, patient witnesses to what *really* matters most in life—love, friendship, and shared visions.

Dedication: Robert B. Davis

Robert Davis' work and writings have focused for more than twenty years on two objectives: trying to improve instructional programs in mathematics and attempting to build an abstract model of human mathematical thought. His writings, particularly *Learning Mathematics: The Cognitive Science Approach to Mathematics Education*, have been a major influence, shaping my theoretical perspective and goals as a classroom teacher, curriculum developer, and researcher. His writings have contributed to my intellectual growth and understanding of fundamental issues in the learning and teaching of mathematics. Bob lived his beliefs—and spent his lifetime working to solve the novel, difficult problems of meeting the social and human needs of students. As a teacher, he shared his vision and wisdom, offering us problems and challenges that aroused our interest. Each of us who was privileged to know him has our own cognitive collages of uniquely wonderful memories of Bob. I am grateful for his friendship, encouragement, and generosity of spirit over the years and consider myself privileged to have known him—to have been able to exchange ideas, share a meal or two, and plan future projects with him. In the past few years, Bob's writings reflected his concern about the growing polarization reflected in the paradigm differences that presently divide those concerned with the learning and teaching of mathematics. Characteristically, his concern was tempered by his optimism and hope for the future. It seems only fitting that a man who spent his life finding ways to gently challenge his students and colleagues has left us yet another problem to focus our energies on:

> Speaking personally, I hope we will pay far more attention in this new era to the paradigm differences that divide those of us who are concerned with the learning and teaching of mathematics. These differences exist, they are extreme, and if we ignore them we shall balkanize an area of intellectual activity that deserves better.

It is to the memory of Robert B. Davis that I dedicate this thesis—a cognitive collage shaped by his ideas and vision—an attempt to take up his challenge.

Declaration

I declare that the material in this thesis has not been previously presented for any degree at any university. I further declare that the research presented in this thesis is my unaided work. The summarized results of the field study quantitative data summarized in Chapter 5 are an expanded version of a paper published in the *British Society for Research into Learning Mathematics 1995 Conference Proceedings,* written with Phil DeMarois and Carole Bernett.

Summary

The fragmentation of strategies that distinguishes the more successful elementary grade students from those least successful has been documented previously. This study investigated whether this phenomenon of divergence and fragmentation of strategies would occur among undergraduate students enrolled in a remedial algebra course. Twenty-six undergraduate students enrolled in a remedial algebra course used a reform curriculum, with the concept of function as an organizing lens and graphing calculators during the 1997 fall semester. These students could be characterized as "victims of the proceptual divide," constrained by inflexible strategies and by prior procedural learning and/or teaching. In addition to investigating whether divergence and fragmentation of strategies would occur among a population assumed to be relatively homogeneous, the other major focus of this study was to investigate whether students who are more successful construct, organize, and restructure knowledge in ways that are qualitatively different from the processes utilized by those who are least successful. It was assumed that, though these cognitive structures are not directly knowable, it would be possible to document the ways in which students construct knowledge and reorganize their existing cognitive structures.

Data reported in this study were interpreted within a multi-dimensional framework based on cognitive, sociocultural, and biological theories of conceptual development, using selected insights representative of the overall results of the broad data collection. In an effort to minimize the extent of researcher inferences concerning cognitive processes and to support the validity of the findings, several types of triangulation were used, including data, method, and theoretical triangulation. Profiles of the students characterized as most successful and least successful were developed. Analyses of the triangulated data revealed a divergence in performance and qualitatively different strategies used by students who were most successful compared with students who were least successful.

The most successful students demonstrated significant improvement and growth in their ability to think flexibly to interpret ambiguous notation, switch their train of thought from a direct process to the reverse process, and to translate among various representations. They also curtailed their reasoning in a relatively short period of time. Students who were least successful showed little, if any, improvement during the semester. They demonstrated less flexible strategies, few changes in attitudes, and almost no difference in their choice of tools. Despite many opportunities for additional practice, the least successful were unable to reconstruct previously learned inappropriate schemas. Students' concept maps and schematic diagrams of those maps revealed that most successful students organized the bits and pieces of new knowledge into a basic cognitive structure that remained relatively stable over time. New knowledge was assimilated into or added onto this basic structure, which gradually increased in complexity and richness. Students who are least successful constructed cognitive structures which were subsequently replaced by new, differently organized structures which lacked complexity and essential linkages to other related concepts and procedures. The bits and pieces of knowledge previously assembled were generally discarded and replaced with new bits and pieces in a new, differently organized structure.

CHAPTER 1 *Thesis Overview*

Say something to us we can learn
By heart and when alone repeat.
Say something!...
Use language we can comprehend.
Tell us what elements you blend...

– Robert Frost, *Choose Something Like a Star*

1.1 Introduction

There is a group of students who have not been the subject of much research to date, those who enroll in undergraduate institutions under-prepared for college level mathematics course work. Remedial (also referred to as "developmental") courses at U.S. colleges and universities are a filter which blocks many students from attaining their educational goals. These students pay college tuition for courses they have taken previously in high school and which do not count for credit towards graduation at most colleges and universities. These courses move along at a pace which many students find impossible to maintain. During each term and in each course, some students succeed, others fail. Dropout rates as high as 50% in the traditional developmental courses have been cited [Hillel, et. al., 1992]. Already over-taxed algebraic skills, combined with time constraints due to unrealistic commitments of full-time enrollment (12 semester hours) and 15 or more hours of outside employment per week on the part of many of these students doom them to yet another unsuccessful mathematical experience. Historically, at the community college of this study, less than 15% of students who initially enroll in a traditional introductory algebra course complete a mathematics course that satisfies general education graduation and/or transfer requirements within four semesters of their enrollment in the developmental program [McGowen, DeMarois, and Bernett, 1995].

1.2 Background and Statement of the Problem

For the most part, students who enroll in the developmental courses could be characterized as victims of "*the proceptual divide*" described by Gray and Tall [1994]. These students have experienced mathematics which "places too great a cognitive strain, either through failure to compress (knowledge) or failure to make appropriate links." They have resorted to the "more primitive method of routinizing sequences of activities–rote learning of procedural knowledge" [Tall, 1994, p 6].

It is not uncommon for the students enrolled in undergraduate remedial mathematics courses to be left with feelings of failure and a belief that mathematics is irrelevant. For these students, mathematics inspires fear, not awe, discouragement, not jubilation, and a sense of hopelessness, not amazement. Why is it that mathematics proves to be so difficult for so many students who attempt rigorous mathematics courses and that they do not succeed? Even many of those who complete three or four years of "rigorous" high school mathematics are unsuccessful in subsequent college-level mathematics courses–only 27 percent of students who enroll in college complete four years, despite the fact that 68 percent of incoming freshman at four-year colleges and universities had taken four years of mathematics in high school [National Center for Education Statistics, 1997].

Many parents, students, and instructors of mathematics believe that there are students "who cannot do mathematics." At a time when our classes increasingly are filled with students that many dismiss as incapable of learning mathematics, we are reminded of Krutetskii's perspective. Thirty-six years ago, in a book for parents, Krutetskii wrote in support of the case of mathematics for all:

> ...generally speaking, the discussion cannot be about the absence of any ability in mathematics, but must be about the lack of development of this ability...Absolute incapability in mathematics (a sort of "mathematical blindness") does not exist... [Krutetskii, 1969a, Vol. II, p. 122].

His description of children's difficulties in learning mathematics also describes the undergraduate students enrolled in developmental algebra courses and the reasons why they are in our remedial courses. He reminds us:

> Don't make a hasty conclusion about the incapacity of children in mathematics on the basis of the fact that they are not successful in this subject. First,...clarify the reason for their lack of success. In the majority of cases, it turns out to be not lack of talent, but a deficiency of knowledge, laziness, a negative attitude toward mathematics, the absence of interest in mathematics, conflict with the teacher, or some other reason, having little to do with ability. Success in removing these causes may bring about great success on the part of the student in mathematics. A common reason for apparent "incapability" in the study of mathematics is that the student does not believe in his abilities as

a result of a series of failures [Ibid., p. 122].

The failure to develop various components of the structure of mathematical abilities identified by Krutetskii are also causes of students' lack of success in addition to the reasons cited. These include the failure to:

- think flexibly;
- develop conceptual links between and among related concepts;
- curtail reasoning;
- generalize;
- modify improper stereotyped learning strategies.

I would add the following which the results of this study suggest underlie and contribute to students' lack of success, in addition to those already cited:

- the qualitatively different ways of constructing and organizing new knowledge and the restructuring of existing cognitive structures;
- inadequate categorization and information-processing skills.

1.3 What skills, when, and for whom?

For many instructors whose teaching responsibilities include large numbers of these students, the question of "What mathematics, when, and for whom?" is the subject of much concern in recent years and is increasingly in need of a response from the mathematics community. Many students do not have as their objective the development of advanced mathematical thinking [e.g in the sense of Tall, 1991a], particularly those who are enrolled in undergraduate developmental mathematics programs. Certainly, for those students who intend to enroll in courses in which they are expected to make the transition to advanced mathematical thinking, a necessary prerequisite is the development of an object-oriented perspective and a high level of manipulative competency [Beth and Piaget, 1966; Dubinsky, 1991; Breidenbach et al., 1992; Cottrill et al., 1996; Sfard, 1995, 1992; Sfard & Linchevski, 1994; Cuoco, 1994; Tall, 1995a].

Undergraduate calculus enrollment in the U.S. has declined 20% in the past five years and increased enrollments in relatively the same percentages in statistics and teacher preparation courses have been reported [Loftsgaarden, et al., 1997]. Given these facts, how appropriately is the present curriculum aligned with the needs of our students? To what degree is the development of an object-oriented perspective necessary for those students who do not have as their goal advanced mathematical thinking; who do not intend to enroll in the calculus course sequence appropriate for future engineers, for those intending to major in mathematics, and for others who need math-intensive programs?

1.3.1 A "Splintered Vision"

Competing visions of what mathematics students should learn have polarized mathematics practitioners and educators, students, their parents, and the community at large. Robert Davis described the position in which we trap students: "There is at present a tug of war going on in education between a 'drill and practice and back to basics' orientation that focuses primarily on memorizing mathematics as meaningless rote algorithms vs. an approach based upon understanding and making creative use of mathematics" [Davis, 1996, personal communication].

These conflicting beliefs and practices were recently cited and the current U.S. mathematics curriculum described as unfocused, "a splintered vision" [Beaton, et. al., 1997]. They are reflected in our mathematics curricular intentions, textbooks, and teacher practices. In comparison to other countries, the U.S. "adds many topics to its mathematics and science curriculum at early grades and tends to keep them in the curriculum longer than other countries do. The result is a curriculum that superficially covers the same topics year after year—a breadth rather than a depth approach." Does this current splintered vision of mathematics really serve the best interests of mathematicians, teachers, students, and the public?

A need for a different vision was argued by Whitehead, who offered the following scathing indictment of algebra as traditionally taught in many classrooms:

> Elementary mathematics... must be purged of every element which can only be justified by reference to a more prolonged course of study. There can be nothing more destructive of true education than to spend long hours in the acquirement of ideas and methods that lead nowhere....[The] elements of mathematics should be treated as the study of fundamental ideas, the importance of which the student can immediately appreciate;...every proposition and method which cannot pass this test, however important for a more advance study, should be ruthlessly cut out. The solution I am urging is to eradicate the fatal disconnection of topics which kills the vitality of our mod-

> ern curriculum. There is only one subject matter, and that is Life in all its manifestations. Instead of this single unity, we offer children Algebra, from which nothing follows...
>
> – Alfred North Whitehead, 1957

The different vision of Algebra called for by Whitehead is still a subject of contention and debate more than sixty years later. Algebra, as envisioned by the U.S. Department of Education, is an essential component of the school curriculum, not a subject which should be eliminated from the curriculum. Recent papers presented at the Algebra Initiative Colloquium set forth principles to guide algebra reform:

- Algebra must be part of a larger curriculum that involves creating, representing, understanding, and applying quantitative relationships.
- The algebra curriculum should be organized around the concept of function (expressed as patterns and regularity).
- New modes of representation need to complement the traditional numerical and symbolic forms.
- Algebraic thinking, which embodies the construction and representation of patterns and regularities, deliberate generalization, and most important, active exploration and conjecture, must be reflected throughout the curriculum across many grade levels.

– The Algebra Initiative Colloquium, 1995

Though the National Council of Teachers of Mathematics proposes the standard "Algebra for All," the *NCTM Curriculum and Evaluation Standards* [1989] fail to clarify what algebra concepts and skills all students should be expected to learn. What do we really mean by "Algebra for All?" In our efforts to make mathematics accessible and attractive to a large number of students, are we, as Al Cuoco worries, "changing the very definition of mathematics?" [Cuoco, 1995].

Terms whose meanings were once commonly understood by those engaged in the practices of mathematics now have different meanings and serve as flashpoints for increasingly vehement discourse. Dialogue based on a common language and definitions has become extremely difficult. As Humpty Dumpty pointed out to Alice in *Through the Looking Glass*: "You see, it's like a portmanteau–there are two meanings packed up into one word." In the absence of mutually agreed-to definitions and accepted meanings, the debate continues among those who favor a “return to basics” and those who are attempting to implement reforms into the teaching and learning of school mathematics, with increasingly high costs for all. Our vision has not only become fragmented, but clouded by emotion. Witness the on-going saga in California where efforts to establish a set of statewide mathematics standards have generated contentious debate and vehemence on both sides. In, 1997, the California State Board of Education revised the K–7 mathematics standards their own appointed commission had worked more than a year to develop. At the heart of the debate is how much emphasis to put on fundamentals such as memorizing multiplication tables and formulas. Appointed Standards commissioners, along with those who support reform initiatives argue that the State Board revisions shifts the focus to a back-to-basics computational approach.

The U.S. government strongly supports the idea of “Algebra for All.” Several recent papers written by staff of the U.S. Department of Education and by U.S. Secretary of Education Richard Riley advocate taking more mathematics courses in high school [National Center for Education Statistics (NCES), 1997]. These documents offer evidence to support the claim that U.S. students wait too long to take Algebra. The assumption that algebra is the key to well-paying jobs and a competitive work force however is challenged by many who claim they succeeded without needing to take Algebra. It requires greater efforts on the part of mathematically-knowledgeable observers to support this assertion with more data and to disseminate the results to the public, as well as to those who teach mathematics in classrooms.

The extent to which problem-solving skills and the use of symbols to mathematize situations are recognized in the workplace frequently go unnoticed by employers as well as by employees [National Center for Education Statistics, 1997]. School mathematics, and algebra in particular, are seen by many as irrelevant, except as a barrier to be gotten past and then forgotten. We urgently need to address the question: What mathematics do we want students to learn? A clearer understanding of the differences and needs of the individual students in our classes must be taken into account in our curricular design and instructional practices. Current practices result in our “building Alban houses with windows shut down so close” some students’ spirits cannot see [Dickinson, 1950].

1.3.2 Flexible Thinking: Interpreting Mathematical Notation

The difficulty facing instructors of remedial undergraduate courses is that of clarifying the reasons for the student's previous lack of success and identifying what precisely is lacking in an individual student's development. Preliminary studies confirmed that one of the difficulties students experience in developmental algebra courses is that of interpreting mathematical notation. They have not learned to distinguish the subtle differences symbols play in the context of various mathematical expressions. What do students think about when they encounter function notation, the minus symbol, or other ambiguous mathematical notation? What are they prepared to notice?

1.4 Theoretical framework

This research is situated within the theoretical framework of current research that suggests that the development of new knowledge begins with perception of objects in our physical environment and/or actions upon those objects [Piaget, 1972; Skemp, 1971, 1987; Davis, 1984; Dubinsky & Harel, 1992; Sfard, 1991, Sfard & Linchevski, 1994; Tall, 1995a]. Perceptions of objects leads to classification, first into collections, then into networks of local hierarchies. Actions on objects lead to the use of symbols both as processes *to do* things and as concepts *to think about*. The notion of *procept*, i.e., "symbolism that inherently represents the amalgam of process/concept ambiguity" was hypothesized by Gray and Tall to explain the divergence and qualitatively different kind of mathematical thought evidenced by more able thinkers compared to the less able [Gray and Tall, 1991a, p. 116].

1.5 Thesis

It is hypothesized that (i) divergence and fragmentation of strategies occur between students of a undergraduate population of students who have demonstrated a lack of competence and/or failure in their previous mathematics courses. In order to explain *why* this phenomenon occurs, it is also hypothesized that (ii) successful students construct, organize, and reconstruct their knowledge in ways that are qualitatively different from those of students least successful and that *how* knowledge is structured and organized determines the extent to which a student is able to think flexibly and make appropriate connections. The inability to think flexibly leads to a fragmentation in students' strategies with resulting divergence between those who succeed and those who do not. These processes of construction, organization, and reconstruction are constrained by a student's initial perception(s) and the categorization of those perceptions which cue selection and retrieval of a schema that directs subsequent actions and thoughts.

1.6 Research questions

A divergence of performance and fragmentation of strategies in elementary grade classrooms have been reported in Russian studies [Krutetskii, 1976, 1969a, 1969b, 1969c, 1969d; Dubrovina, 1992a, 1992b; and Shapiro, 1992] and in the studies of Gray and Tall [1994, 1993, 1992, 1991b, 1991c], and Gray, Tall, and Pitta [1997]. This study investigated the nature of the processes of knowledge construction, organization, and reconstruction and the consequences of these processes for undergraduate students enrolled in a remedial algebra course. Strategies students employed in their efforts to interpret and use ambiguous mathematical notation and their ability to translate among various representational forms of functions were also subjects of study. Given a population of undergraduate students who have already demonstrated a lack of competence or failure previously, the main research questions addressed are:

- does divergence and fragmentation of strategies occur among undergraduate students enrolled in a remedial algebra course who have previously been unsuccessful in mathematics?
- do students who are more successful construct, organize, and restructure knowledge in ways that are qualitatively different from the processes utilized by those who are least successful?

Related questions addressed students' ability to think flexibly, recognize the role of context when interpreting ambiguous notation, and develop greater confidence and a more positive attitude towards mathematics. The study examined whether students classified as 'less able' and/or 'remedial,' could, with suitable curriculum:

- demonstrate improved capabilities in dealing flexibly and consistently with ambiguous notation and various representations of functions?
- develop greater confidence and a more positive attitude towards mathematics?

1.7 Design and Methodology

One aim of this research was to extend the classroom teaching experiment [Steffe & Cobb, 1988; Steffe, von Glaserfeld, Richards and Cobb, 1983; Confrey, 1995, 1993, 1992; Thompson, 1996; 1995] to students at undergraduate institutions enrolled in a non-credit remedial algebra course. This course is prerequisite for the vast majority of U.S. college mathematics courses. The subjects of study were twenty-six students enrolled at a suburban community college in the Intermediate Algebra course. A reform curriculum was used, with a process-oriented functional approach which integrated the use of graphing calculator technology.

Research for this dissertation included two preliminary studies: a broad-based field study (n = 237) and a classroom-based study (n = 18) at the Chicago northwest suburban community college which was also the site of the main study. The quantitative field study was undertaken in order to develop a profile of undergraduate remedial students and to characterize some of the *prior variables* they bring to the course, such as their attitudes and beliefs. Classroom-based preliminary studies were conducted so that a local student profile could be developed and prior variables identified, which could be compared with those of the broader-based field study. A preliminary classroom-based qualitative study also investigated students' ability to deal with ambiguous mathematical notation.

The main study (n=23) included both quantitative and qualitative components. Data was collected which focused on two groups of extremes: the most successful and least successful students of those who participated in the study. Students' *concept maps,* i.e., external visual representations of a student's internal conceptual structures at a given moment in time, were used to document the processes by which the most successful and least successful students construct, organize, and reconstruct their knowledge and to provide evidence of how students integrate new concepts and skills into their existing conceptual frameworks. They also reveal the presence of inappropriate *concept images* (in the sense of Tall and Vinner, 1981) and connections.

The accumulated data reported in this study was interpreted within a multi-dimensional framework based on cognitive, sociocultural, and biological theories of conceptual development, using selected insights representative of the overall results of the broad data collection of this research. In an effort to minimize the extent of researcher inferences concerning cognitive processes and to support the validity of the findings several types of triangulation were used, including data, method, and theoretical triangulation [Bannister et. al., 1996, p. 147]. Profiles of the students characterized as most successful and least successful were developed based on analysis and interpretation of the triangulated data.

1.8 General Conclusions

The most successful students construct and organize new knowledge and restructure their existing conceptual structures in ways that are qualitatively different from those of the least successful students. The divergence and fragmentation of strategies over time of undergraduate remedial students were documented, both quantitatively and qualitatively. Qualitative differences were found that suggest that the most successful students:

- experienced growth in understanding and in competence to a far greater extent than did the least successful, who experienced almost no growth in understanding or improvement in their mathematical abilities.
- constructed and organized new knowledge into a basic cognitive structure that remained relatively stable over time.
- assimilated new bits and pieces of knowledge into this basic structure, generally enriching the existing structure(s) and by accommodation which resulted in a restructuring of existing cognitive structures over time.
- focused on qualitatively different features of perceived representations than did the least successful students.
- used classification schemes which were qualitatively different from those used by the least successful students.
- improved in their ability to deal flexibly with the ambiguity of notation.
- improved in their ability to translate among various representations of functions during the semester.
- improved in their ability to reverse their train of thought from a direct process to its reverse process.
- demonstrated an ability to curtail reasoning in a relatively short period of time.

- exhibited a consistency of performance in handling a variety of conceptual and procedural tasks stated in several different formats and contexts, using various representational forms.

- were able to demonstrate they had developed *relational* understanding, i.e., they were able to make connections with an existing schema which resulted in a changed mental state which gave them a degree of control over the situation not previously demonstrated, accompanied by a change in feeling from insecurity to confidence.

Least successful students, on the other hand

- replaced their existing cognitive structures with new structures. They retained few, if any of the bits and pieces of knowledge previously assembled in the new, differently organized structure.

- were constrained by their inefficient ways of structuring their knowledge and inflexible thinking. Caught in a procedural system in which they were faced with increasingly more complex procedures, they increasingly experienced frustration and cognitive overload.

- demonstrated a lack of appropriate connections which contributed to their inability to flexibility recall and select appropriate procedures, even when they had these procedures available to them.

- were unable to curtail their reasoning within the time span of the semester in many instances.

- were inconsistent in handling a variety of conceptual and procedural tasks stated in several different formats and contexts, using various representational forms.

Other findings indicate that:

- the initial focus of attention cues the selection of different cognitive units and retrieval of different schemas by the two groups of students of the extremes.

- there were generally positive changes in nearly all students' beliefs about their ability to interpret mathematical notation, interpret and analyze data, and to solve a problem not seen previously. There was also a positive change in attitude about the use of the graphing calculator to better understand the mathematics and in the willingness to attempt a problem not seen previously.

1.9 Thesis Organization

This thesis consists of nine chapters, a bibliography, and appendices.

Chapter 1 contains an overview of the thesis and includes: a brief introduction and background description; a statement of the problem; a brief description of the theoretical framework on which the study is based; the thesis and the main research questions; an overview of the methodology and design of the study; and a summary of the conclusions. This synopsis of the dissertation concludes the chapter.

Chapter 2 is a general literature review. The main research topics reviewed include: the nature of cognitive structures and their organization; the processes of knowledge construction; relevant theories of cognitive development, issues of representation, and current issues of knowledge acquisition.

Chapter 3 describes the researcher's theoretical perspective and how this perspective is situated among past and current research. A theoretical model of the processes of knowledge construction is presented, situated among other major models previously developed, together with the main theses and research questions. A rationale for the use of concept maps and corresponding schematic diagrams as tools of analysis to document the nature of students' processes of construction, assimilation, and accommodation is presented.

Chapter 4 describes the methodology and key components of the methods used to collect and analyze the data reported in this study. The methodology and methods employed in this study are situated within the theoretical framework of constructivist extended teaching experiments adapted to the study of undergraduate students enrolled in a remedial Intermediate Algebra course.

Chapter 5 describes the preliminary studies. A description of the subjects of the study, the instruments used, a summary of the data, and observations resulting from the analysis of the data are presented. The preliminary studies include a broad-based field study, a local, classroom-based quantitative study and a qualitative classroom study which examined students' difficulties interpreting ambiguous notation and in reconstructing their existing concept images. The chapter concludes with a summary of and conclusion about the findings of the preliminary studies. Modifications made in the data collection instruments and methods of analysis prior to undertaking the main study are described.