

**REASSESSING THE FUNDAMENTALS**  
**On the Evolution, Ages and Masses of Neutron Stars**

**Bülent Kızıltan**

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*Reassessing the Fundamentals:  
On the Evolution, Ages and Masses of Neutron Stars*

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## Abstract

*REASSESSING THE FUNDAMENTALS:*  
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NEUTRON STARS

by

Bülent Kızıltan

The evolution, ages and masses of neutron stars are the fundamental threads that make pulsars accessible to other sub-disciplines of astronomy and physics. A realistic and accurate determination of these indirectly probed features play an important role in understanding a very broad range of astrophysical processes that are, in many cases, not empirically accessible otherwise.

For the majority of pulsars, the only observables are the rotational period ( $P$ ), and its derivative ( $\dot{P}$ ) which gives the rate of change in the spin. I start with calculating the joint  $P$ - $\dot{P}$  distributions of millisecond pulsars for the standard evolutionary model in order to assess whether millisecond pulsars are the unequivocal descendants of low mass X-ray binaries. We show that the  $P$ - $\dot{P}$  density implied by the standard evolutionary model is inconsistent with observations, which suggests that it is unlikely that millisecond pulsars have evolved from a single coherent progenitor population.

In the absence of constraints from the binary companion or supernova remnant, the standard method for estimating pulsar ages is to infer an age from the rate of spin-down. I parametrically incorporate constraints that arise from binary

evolution and limiting physics to derive a “modified spin-down age” for millisecond pulsars. We show that the standard method can be improved by this approach to achieve age estimates closer to the true age.

Then, I critically review radio pulsar mass measurements and present a detailed examination through which we are able to put stringent constraints on the underlying neutron star mass distribution. For the first time, we are able to analyze a sizable population of neutron star-white dwarf systems in addition to double neutron star systems with a technique that accounts for systematically different measurement errors. We find that neutron stars that have evolved through different evolutionary paths reflect distinctive signatures through dissimilar distribution peak and mass cutoff values. Neutron stars in double neutron star and neutron star-white dwarf systems show consistent respective peaks at  $1.35 M_{\odot}$  and  $1.50 M_{\odot}$ , which suggest significant mass accretion ( $\Delta m \approx 0.15 M_{\odot}$ ) has occurred during the spin up phase. We find a mass cutoff at  $2 M_{\odot}$  for neutron stars with white dwarf companions which establishes a firm lower bound for the maximum neutron star mass. This rules out the majority of strange quark and soft equation of state models as viable configurations for neutron star matter. The lack of truncation close to the maximum mass cutoff suggests that the  $2 M_{\odot}$  limit is set by evolutionary constraints rather than nuclear physics or general relativity, and the existence of rare super-massive neutron stars is possible.

To my family...

...Aileme

## Acknowledgments

It's been truly an amazing journey. I must say it almost hurts to think that my life as a student will soon come to an end. Despite the obvious disadvantages of being a student, I shall not complain. My life in Santa Cruz set the bar too high, and it will be difficult to even imagine a better place to live...

Stress is a constant in student life regardless whether one lives in Santa Cruz or not. Throughout my life as a student, especially during graduate school, I have observed many cases where advisors were the source of stress. I am fortunate that this has not been my case. Foremost, I thank my advisor, Steve Thorsett, for being so supportive throughout my graduate endeavor. He is one of the most positive academics I've ever met. His well balanced guidance and encouragement earned me, among other traits that I consider invaluable, the degree of "the most travelled graduate student". I had the opportunity to interact with many bright minds during these trips which resulted in fruitful cross pollination that gave me some of my best ideas. I have been inspired by Steve's intuitive approach to the mathematical aspects of astrophysical problems, which eventually triggered my interest in furthering my training and understanding in statistics. I thank Steve also for being so accessible, and making time even when, as the dean, he was rushing from one meeting to another.

I am thankful to my qualifying exam and dissertation reading committee. Greg Laughlin who has been there as a career mentor and a neighbor down the corridor; Enrico Ramirez-Ruiz who has always shown the desire to swing by and

give an extra nudge; and Ingrid Stairs for her willingness to travel all the way from Vancouver to serve in my committee. I also thank Josh Grindlay for his generous hospitality at Harvard and for the stimulating discussions that led me to question some of the very basics of millisecond pulsar evolution and accretion physics. I acknowledge Athanasios Kottas who has taken delicate steps to introduce the foundations of Bayesian statistics. It turns out learning from peers cannot be rivaled by others. Santa Cruz could not have been the top ranked center for astronomy research without it's students. I thank all of my peers from whom I have also learned so much.

I thank everybody who contributed to make the department the “best” educational environment any junior researcher can hope for. Particularly I thank Sandra Faber for leading the way in pushing the envelope to make the department not only a nurturing environment for students but also a center for excellence. She has been truly a role model. I also thank Raja Guhatakurta who has been involved with every graduate student, including myself, at a level that exceeded the call of duty.

This journey could not have been a success without the constant and unwavering support of my family. *Can'larım... Her zaman yanımda olduğunuz için, sevginizle karşılıksız sevmeyi öğrettiginiz için, ve de “siz” olduğunuz için... Çok teşekkürler.* I cannot thank them enough for always being there for me.

I am afraid I will never let go of the hunger for inquiry and the humbling experience of learning. If this is being a student, there I will remain to be...

**Contents:** The text of this dissertation includes reformatted and in parts updated versions of the following previously published material: [Kiziltan & Thorsett \(2009a\)](#), [Kiziltan & Thorsett \(2010b\)](#) and [Kiziltan et al. \(2010b\)](#). Parts of the texts from [Kiziltan & Thorsett \(2009b\)](#), [Kiziltan & Thorsett \(2009c\)](#), [Kiziltan & Thorsett \(2010a\)](#), [Kiziltan et al. \(2010a\)](#), and [Kiziltan \(2010\)](#) have been also used in the summary, discussion and conclusions sections. As the lead author, I am responsible for the scientific direction, content, analysis and the text in the above mentioned articles. NASA-GLAST/Fermi funding and National Science Foundation grant AST-0506453 supported some of the research presented here.

**Cover image credit:** NASA

It ain't what we don't know that gets us into trouble.

It's what we know for sure that just ain't so.

...Mark Twain

# Part I

## PRELUDE



# Preface

## Ph.D.

It is almost time to finally get a *Philosophiae Doctor*. It should be pointed out that the “philosophy” in the highest academic title that can be earned has nothing to do with the scientific discipline “Philosophy”. It rather translates literally into “*the love of wisdom*”, or may loosely mean “*the pursuit of in-depth knowledge*”. It is my love of wisdom that pushes towards pursuing the in-depth knowledge about the process of “knowing”; and it can only be fitting that a scientist builds a consistent stands by which (s)he initiates and carries out such a pursuit.

Therefore, a Ph.D. dissertation without saying a few words on the philosophy of this pursuit would be too dry, if not incomplete.

## Is Simplicity Simple?

To an observing eye it is almost impossible to miss the two schools of thought prevalent among scientists: The thinking minds of one school have a grandiose approach to scientific problems. They have an instinct to unify everything in one grand scheme, whereas the diligent minds that belong to the other camp appreciate complexity and sophistication (see [Craig, 1998](#)).

The former approach may have gained popularity with the emergence of modern science, particularly with the paradigm shift Einstein has triggered. While

Einstein wanted a simple, unified theory from which complexity would emerge logically, for Bohr, quantum mysteries such as the dual wave-and-particle nature of light reflected the richness of a complicated universe.

It was perhaps when Einstein came to the realization that his postulation of the cosmological constant was his “biggest blunder”, he gradually adapted a different approach. The same Einstein who had a “...humble admiration of the illimitable superior spirit who reveals himself in the slight details we are able to perceive with our frail and feeble minds”, also belittled the efforts that ultimately sought to discover these details by saying “Any intelligent fool can make things bigger and more complex... It takes a touch of genius - and a lot of courage to move in the opposite direction”. Was Einstein overindulging in the fame and pride that came with his success? Was this a manifestation of an over-reliance on human intuition? Or was this merely a vocalization of what the limited human perception desires? Or rather a deeper understanding of the ultimate truth...?

It may be possible to harbor this dichotomy within oneself without suffering from the shortcomings of inconsistency.

I observe that in many fields, astronomy not withstanding, many get overcome by the desire to be “the” person that reaches a higher level of understanding who, then, prematurely push in favor of a “unified” model. Perhaps this is a natural state of the human mind which is wired as such to gradually perceive the more sophisticated. While this remains to be an intermediary step in the direction of the next paradigm, a premature push towards unification may, and will, hamper the efforts of fully exploring the full complexity.

## **Complexity May Be Simple, But Not Simpler...**

I oppose to belittling the efforts that provide a fuller insight into how detailed and sophisticated natural processes can be. At the brink of a paradigm shift, over simplification will lose details necessary to reach a higher level of understanding. It is crucial to reach a sufficiently clear awareness and understand the complexity in it's full glory in order to reach a higher state of perception. Einstein was ultimately lucky to live in a period when the details were emerging which carried him to the conception of general relativity. While his intellect and power of "imagination" were indisputably clear, it can be argued that he would have never been able to put the pieces together without the efforts of many others who embraced diversity and polished the details.

However, it would be equally impossible to shift the paradigm with a mind that is drowned in the details of the puzzle (see [Kuhn, 1970](#)). The impossibility of reaching a perception that is out of the box can be made possible only if the pieces of the puzzle have come to existence.

## **Simply Sophisticated Leads to Alternative**

That being said, nowhere during the process that produced what is included in this dissertation, did I necessarily aim to find alternatives to the conventional that is widely accepted as "canonical". One of the initial triggers was, perhaps, the desire to provide a convincing scheme by which the qualitative perceptions

can be justified with quantitative evidence (see [Popper, 1992](#)). My goal was to go beyond the qualitative assessments which were considered informed “common sense”, and use rigorous tools to quantify the problem. The goal was to see whether the numbers were in fact confirming the conventional picture, or were they rather telling a different story (see [Lakatos & Feyerabend, 1999](#)).

As a junior scientist, I have the luxury of not being invested too much in any of the scenarios that I was investigating. This gave me the intellectual freedom to adopt a rather agnostic approach to the problems at hand. It turns out that, in all cases, the numbers lead me to an alternate direction beyond the conventional road.

## **Scientific Dogma or Dogmatic Science?**

Here, with this dissertation, it is my goal to demonstrate that we are still away from appreciating the picture in its full diversity. While we lack a completed jigsaw puzzle, the pieces are most likely already in existence and accessible. The ingredients of the final glue to complete this jigsaw puzzle will not be without the next generation numerical tools that will assist the efforts of putting these problems into a broader context. Especially, in data driven fields such as astronomy and astrophysics, it is imperative to go beyond preconceptions, and let the data speak for itself. While intuition is perhaps an invaluable tool a scientist cannot do without, an arrogant over-reliance on intuition will only dogmatize our stand. While in the modern era we appear to be diligent in resisting to (pseudo) scien-

tific dogma, how much of that diligence remains alive and is directed also towards dogmatic science?

I cannot help but ask: Can a young clerk working at a patent institute successfully go through today's peer review process and publish his short paper that mainly consists of "gedanken" experiments in any prestigious journal? The question begs the answer that we are, perhaps, still far from the next paradigm shift!

Santa Cruz, California

*Bülent Kızıltan*

# Chapter 1

## INTRODUCTION

### 1.1 Background

#### 1.1.1 Evolution of Millisecond Pulsars

Millisecond pulsars are commonly believed to be descendants of normal neutron stars that have been spun-up and recycled back as radio pulsars by acquiring angular momentum from their companion during the low-mass X-ray binary (LMXB) phase ([Alpar et al., 1982](#); [Radhakrishnan & Srinivasan, 1982](#)).

There are about  $\sim 20$  high confidence nuclear or accretion powered (see [Table 1.1](#)) millisecond X-ray pulsars (MSXPs) which are thought to be the progenitors of millisecond radio pulsars (MSRPs) ([Wijnands & van der Klis, 1998](#)). These millisecond X-ray pulsars may become observable in radio wavelengths once accretion ceases, or the column density of the plasma from the fossil disk around the neutron star becomes thin enough to allow vacuum gap formation that leads

Table 1.1. Accretion and nuclear powered pulsars

$\nu_{\text{spin}}$ [Hz]	Pulsar
619	4U 1608–52
601	SAX J1750.8–2900
598	IGR J00291+5934
589	X 1743–29
581	4U 1636–53
567	X 1658–298
549	Aql X-1
552	EXO 0748–676
524	KS 1731–260
435	XTE J1751–305
410	SAX J1748.9–2021
401	SAX J1808.4–3658
377	HETE J1900.1–2455
363	4U 1728–34
330	4U 1702–429
314	XTE J1814–338
270	4U 1916–05
191	XTE J1807.4–294
185	XTE J0929–314

Note. — Accretion and nuclear powered pulsars. The millisecond pulsar progenitor seeds used to construct the cumulative synthetic MSRP population for the observed  $P_{\text{MSXP}}$ .

to the production of coherent radio emission. Towards the end of the secular LMXB evolution, as accretion rates fall below a critical value above which detection presumably may be hampered due to absorption or dispersion ([Thompson et al., 1994](#)), the neutron star can re-appear as a millisecond radio pulsar.

Although the connection between LMXBs and millisecond radio pulsars has been significantly strengthened after the discovery of quasi-periodic kHz oscillations and X-ray pulsations in some transient X-ray sources ([Galloway et al., 2002](#),

2005; Markwardt et al., 2002; Wijnands & van der Klis, 1998), no radio pulsations from millisecond X-ray pulsars have been detected so far (Burgay et al., 2003).

At the end of the recycling process the neutron star will reach an equilibrium period (Bhattacharya & van den Heuvel, 1991) which is approximated by the Keplerian orbital period at the Alfvén radius (Ghosh & Lamb, 1992):

$$P_{\text{eq}} \sim 1.9 \text{ ms } B_9^{6/7} \left( \frac{M}{1.4 M_\odot} \right)^{-5/7} \left( \frac{\dot{m}}{\dot{M}_{\text{Edd}}} \right)^{-3/7} R_6^{16/7} \quad (1.1)$$

where  $B_9$  and  $R_6$  are the neutron star surface magnetic dipole field and radius in units of  $10^9$  G and  $10^6$  cm respectively. The Eddington limited accretion rate  $\dot{M}_{\text{Edd}}$  for a neutron star typically is  $\sim 10^{-8} M_\odot \text{ yr}^{-1}$  above which the radiation pressure generated by accretion will stop the accretion flow. This equilibrium period combined with the dominant mechanism for energy loss delineates the subsequent kinematics of the spun-up millisecond pulsar. The magnetic dipole model then implies a “spin-up region” ( $\dot{P} \sim P_0^{4/3}$ ) (see Arzoumanian et al., 1999) on which the recycled neutron stars will be reborn as millisecond radio pulsars. At the end of the active phase, millisecond X-ray pulsars accreting with  $\dot{m}$  and spinning with  $P_{\text{eq}}$ , presumably transition into a millisecond radio pulsars with an initial spin period of  $P_0 \sim P_{\text{eq}}$ .

In the standard spin-down model, the millisecond radio pulsar evolution is driven by pure magnetic dipole radiation, i.e. braking index  $n = 3$  in vacuum (see Lorimer & Kramer, 2004; Manchester & Taylor, 1977). Alternative energy loss mechanisms such as multipole radiation or gravitational wave emission, especially



during the initial phases of the reborn millisecond pulsars, have been suggested by several authors (Bildsten, 1998; Krolik, 1991) but have yet to be observationally corroborated. Advanced Laser Interferometer Gravitational Wave Observatory (LIGO) will be able to probe the frequency space at which millisecond pulsars are expected to radiate gravitational waves, thereby putting stringent constraints on the micro physics of millisecond pulsars.

### 1.1.2 Ages of Millisecond Pulsars

An accurate determination of pulsar ages plays a critical role in our understanding of advanced stages of stellar evolution, supernova explosions and remnants, white dwarf (WD) atmospheres and cooling models, binary evolution, planet formation around compact objects, and pulsar evolution in general.

Typically, pulsar ages are estimated by calculating the amount of energy lost during their spin-down. Consequently, the spin-down age of a pulsar can be formulated as:

$$\tau = \frac{P}{(n-1)\dot{P}} \left[ 1 - \left( \frac{P_0}{P} \right)^{n-1} \right] \quad (1.2)$$

where the period ( $P$ ) and the spin-down rate ( $\dot{P}$ ) are the two main observables acquired by pulsar timing measurements. In the standard approach, the unknown initial spin period ( $P_0$ ) of the pulsar is assumed to be much smaller ( $P_0 \ll P$ ) than the observed period. The dominant energy loss mechanism is analytically captured by the braking index, which is  $n = 3$  for pure dipole radiation and is implicitly adapted by the characteristic age  $\tau_c$ . The age of a pulsar can then be