Decision Making in Uncertain Situations: An Extension to the Mathematical Theory of Evidence

by

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Fábio Campos

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The danger of dreaming is to achieve the dreams.

This work is a fruit of an old dream.

I dedicate it:

to everyone who chases their dreams,

to Márcia, my dream,

and to my father Ryam (in memoriam) who, from my earliest years taught me this and the appreciation for research and experimentation in all fields of life.

Acknowledgements

Every sacrifice or effort to do what one likes becomes, in the end, a great delight.

Thus, if there is someone who deserves the most thanks for the innumerable nights and weekends without me, this person would be my wife, Márcia, who gave up the most for this work, followed closely by the rest of my family, in particular, my mom, Hermínia, my grandmother, Dulce, and my friends.

I also could not forget to thank:

– my friends from the Informatics Center - UFPE, in particular those from the GrECo (Computacional Engineering Group), companions of many night-long journeys of studies and debates to solve the most diverse computer problems;

– to my graduate tutor, Prof. Sérgio Cavalcante, who always thinks of himself as a problem solver, overcoming all the obstacles to the development of the ideas presented here, believing and supporting this cool madness that allowed ourselves create a work in an area totally diverse from which that which we initially conceived, with all the risks associated;

– to André Leite (Graduate student in the Master's program) who patiently helped with the revising of many details;

– and to Prof. Fernando Campello de Souza an unconditional supporter of all my scientific initiatives, both academic and entrepreneurial, to whom I owe a significant portion of my formation, and more precisely that part which has helped me to create the present work.

What it is the reality around us but the perception that we extract from the probabilities?

What it is the "known universe" but a huge amount of empty space, populated here and there by regions of probable encounters among subatomic particles whose behavior we can perceive as solids, liquids and gasses?

And indeed what are people, things, ideas and thoughts but probabilistically-coded information, being constantly updated for the next event, implying in things like their progress, movement, elaboration, decrease and growth?

That is, everything, in fact, consists of an eternal play of the divine dice...

— FÁBIO CAMPOS (2004)

Contents

List of Tables

Foreword

The core of this work was presented to the Computer Science Graduation Program of the Informatics Center at “Universidade Federal de Pernambuco – UFPE” as one of the requirements for the obtention of a PhD in Computer Science.

Before we pass to the content of the book itself it is important to explain the motivation behind it. The work from which this book is a consequence began in the Computation Engineering and Embedded Systems area and finished as a PhD thesis in the area of Uncertain Probabilities [7].

Our initial project goal was to achieve “A Methodology for Embedded Systems Design”, where we intended to study knowledge representation derived from the user inputs, the representation of uncertainty and conflict in a knowledge-base, and a mapping algorithm that would have allowed us to associate users’ requisites to the project orientations suggested by the knowledge-base.

To implement this project, there was the need to adopt a knowledge representation model able to consider both the uncertainty and the conflict present in the inputs supplied by the users and those existing internally in the knowledge-base. Among the several available formalisms for the representation and combination of this kind of knowledge, we chose, for technical matters, mainly the kind of sample space and the nature of the uncertainties involved, Evidence Theory, also known as the Dempster-Shafer Theory, or the Mathematical Theory of Evidence.

When we began to study the Dempster-Shafer Theory, we verified that theoretical problems could cause counter-intuitive results or limit the modeling power of this formalism, making the practical application of this theory, in our particular application, excessively complicated. In the search of a better understanding of the problem, we verified that these issues had been already suggested by one of the creators of the theory, Glenn Shafer, in his seminal article presenting the theory itself, dated 1976 [45]. In this article, Shafer explain that counter-intuitive results would occur when the evidence to be combined had belief concentration in disjoints events and a only small quantity of belief in a common event. Although there have been several attempts to solve this behavior, only partially acceptable solutions have been achieved. Through a simple but ingenious solution, we managed to solve this classical problem, also obtaining as a by-product a more epistemic modeling for combining all kinds of evidence (that is, not just in cases where the subject is known to show counter-intuitive results before hand). This extension of the Dempster-Shafer Theory allows its use in a larger range of applications, as well as providing an improvement in the modeling of the original range of applications.

Thus, the solution of this classical problem, together with its implications, became the object of my PhD thesis. When we introduced this solution to the Decision Theory Group of UFPE, the questions and discussions raised helped us see that the conceptual implications of the solution had ramifications rather larger than those we had glimpsed initially; and this led us to what became the main object of this book, an extension to the Mathematical Theory of Evidence, able to treat both objective and epistemic uncertainties, allowing decision making in uncertain situations [8].

Organization of the Book

The initial chapter introduces the main goal of this book and provides the establishment of a framework that allows us to treat either the subjective as the objective uncertainties, solving the classical problems of the theories that deal with evidence, as well as eliminating the dichotomy of treatment between these two kinds of uncertainties. This is followed by the justification and motivation which provides the foundation for a work of this kind.

The following chapter describes the state-of-the-art and related works. The first section of this chapter is devoted to the subject of subjective uncertainty showing its philosophical basis and the need to consider this kind of uncertainty. It continues with an explanation of knowledge representation and combination, in Section 2.3, where the several kinds of imperfections to which information can be subjected are shown. Section 2.4 follows with an explanation of several interpretations of "probability". In this section, several different nuances of interpretation are clarified, which also provides the basis for the title of the book: "Decision Making in Uncertain Situations – An Extension to the Mathematical Theory of Evidence", since it deals with beliefs and not with "classic probabilities".

Chapter 3 explains the Dempster-Shafer Theory, the conceptual and formal basis upon which the extension presented in this book is built.

Chapter 4 proposes an including beliefs theory. Section 4.1 clarifies imperfections of the Dempster-Shafer Theory, and Section 4.2 proposes a solution, through the adoption of a new rule of evidence combination and the associated conceptual framework (vide Section 4.2.2). This section concludes with a validation of the extension to the theory (Section 4.2.3).

Chapter 5 presents a case study of a possible application of the extension. This example elucidates the importance of the quantification of the uncertainty embodied in the results using real financial data from the Brazilian market.

Concluding the book, Chapter 6 summarizes the results and conclusions obtained, as well their practical and conceptual implications.

1 Introduction

This work extends the Mathematical Theory of Evidence, also known as the Dempster-Shafer Theory, through the adoption of a new rule for the combination of evidence and a companion set of concepts. This extension solves the counter-intuitive problems illustrated in the original theory, extends its power of expression and allows the representation of uncertainty in the results.

The representation of uncertainty implies the possibility of its use in decision-making and also makes explicit the relationship between the numeric results achieved and the results from classical probability theory.

The main problem addressed by this book is how to model and combine bodies of knowledge while maintaining the representation of the unkowledge and of the conflict among the bodies.

This is a problem with far-reaching applications in many knowledge segments, in particular for the field of artificial intelligence. It must be kept in mind that knowledge based systems depend on algorithms able to relate the inputs of a system to a correct answer coming out of the knowledge-base, and both the inputs and the knowledge-base are subject to information imperfections caused by the unknowledge and the conflict.

There are several formalism to deal with knowledge representation and combination, among them the Mathematical Theory of Evidence or Dempster-Shafer Theory. This theory has received considerable attention since it is suitable to represent naturally a wide range of situations [42] and is more general than the widely-used Bayesian Theory, which relates to a specific case of the Theory of Evidence [25].

However, the Mathematical Theory of Evidence exhibits two major limitations: the possibility of arriving at a counter-intuitive result; and the lack of representation of the degree of subjective uncertainty in the results [21]. This latter limitation can permit the combination of conflicting bodies of knowledge resulting in the same numeric values as a combination of bodies with no conflict at all. These limitations narrow the range of applications of the theory while at the same time demand the elimination of some bodies of knowledge which could, in other ways, contribute to the construction of the knowledge. It should be noted that the original theory presents as a principle that if a source of knowledge is consulted, the evidence coming from it must be taken into consideration, even if only to increase the degree of uncertainty in the combination result (the "specialist" concept) [45].

Since the formalization of the theory by Shafer in 1976 [45], several attempts have been made to identify the cause and solution of counter-intuitive behavior resulting from several particular combinations. For example, Reference [67] represents a collection of papers in which such authors as Smets, Yager and Zadeh attribute counter-intuitive behavior to the normalization step of Dempster's Rule of Combination. Based on this premise several other rules of combination have been tried, which eliminate the normalization step (vide Section 4.2.3 for references and authors) in an attempt to solve at least part of the counter-intuitive behavior of the original rule. The adoption of these rules, however, leads to important side effects that also result in counter-intuitive behaviors in some specific situations.

Taking all of this into account, the main objective of this book is to present an extension to the Theory of Dempster-Shafer by the development of a conceptual basis and the adoption of a new rule of evidence combination able to increase significantly the power of expression of the theory, solve its counter-intuitive behavior and implement a way to represent, in numeric terms, the uncertainty that comes from unkowledge and conflict among the bodies of evidence.

1.1 Justification

The "Uncertainty concept" has been being one of the most elaborated scientific concepts in recent times [26]. In general, uncertainty in a "problem-situation" emerges whenever the information pertinent to the situation is deficient in some aspect. This deficiency can be caused by a piece of information which is incomplete, imprecise, contradictory, vague, non-reliable, fragmented, or deficient in some other way, giving origin to several kinds of uncertainties [25]. These kinds of uncertainties can be classified in two large groups, objective uncertainty and subjective uncertainty, giving origin to what is known as the "dual nature of uncertainty" defined by Helton in 1997 [23]. However, just recently the scientific and engineering community has started to recognize the usefulness of establishing definitions and treatment models for different kinds of uncertainties [21]. This has been motivated by the extensive development of mathematical analysis since the end of the nineteenth century (by Cantor, Lebesgue, Kolmogorov and De Finneti, just to list some). The applicability of this in every-day situations has been facilitated greatly by the extraordinary computational power easily available today. As soon as systems were able to work with complex analysis, it became easy to discover the limitations of the Bayesian Theory in representing and treating the whole range of uncertainties. A primary motivation to study reasoning subjected to uncertainty is to be able to reach decisions when facing otherwise non-conclusive evidence [54].

The dual nature of uncertainty has been assigned to a concept named "Belief", different from the concept of "Probability", that is used more in the sense of traditional objective (the frequency approach) and subjective probabilities. "Objective probability" (a frequentist approach to probability) can be understood as the probabilistic knowledge obtained on the basis of the relative frequency of the occurrence of events in a long sequence of independent experiments; and "subjective probability" (or Bayesian Probability) as referring to the alteration or the conditioning of a previous probability measure, in function of new evidence or observation.

On the other hand, "Belief" relates to conviction, whether or not this conviction is supported by the concepts of traditional probability. Belief is not necessarily connected to decision-making or betting contexts. It is a cognitive process *per se* [50]. Belief aims to model and quantify both objective and subjective certainties, induced in us by the evidence [50].

Some criteria to support Belief are [53]:

- Faith: a hypothesis is believed because the person who established it is dependable.

- Reasonability: the hypothesis is accepted if it agrees with previously established beliefs.

- Success of Prediction: the hypothesis is believed when it is able to foresee the behavior of events not yet observed.

The Dempster-Shafer Theory is able to formalize these two different kinds of uncertainty simultaneously [25], while the Bayesian Theory models naturally only the objective uncertainty. Because of this the Dempster-Shafer Theory was chosen as the basis for this work.

1.2 Motivation

There are important reasons, practical and theoretical, to study the representation, combination and comparison of beliefs. Humans frequently reason in both objective and subjective terms (understanding this as the quantitative and qualitative aspects of reasoning) [66], thus, the representation of reasoning in the form of beliefs seems to be a more realistic model of uncertainty, particularly when the available information is limited[1]. As there exist situations that demand actions to be taken based not only on the available knowledge, but also on what is recognized as not known, there is a need for the development of non-classic logic, that is, a logic conceived and put into practice to capture particular kinds of reasoning, such as reasoning about beliefs, probabilistic reasoning or default reasoning [31]. These types of logic are not conceived because the classical paradigm supplies wrong answers, but because certain questions cannot be expressed easily, naturally or efficiently (from the computational point of view) [31]. These types of logic result in a meta-theoretical extension of the power of expression of classical logic, since it is not able to talk about ignorance, since reasoning about ignorance results in an odd consequence, non-monotonicity[2] [31], and classical logic is monotonic by principle. Additionally, if belief functions are thought of as generalizations of probability functions (as will be shown below), then the understanding of the mathematical behavior of the beliefs becomes as important as the probability study itself.

Considering these aspects, the motivation for the utilization of the Dempster-Shafer Theory and its extension presented in this work, in the beliefs modeling, arose not just from the technical characteristics of the theory, but also from its wide range of practical applications, mainly in the last 10 years [42], indicating a high degree of experimentation, maturity, and relevance. Practical applications of the Dempster-Shafer Theory can be found in all the sciences, from human to exact. Some examples include image processing, voice recognition, specialist systems, knowledge-based systems, robotics, decision support systems [48], fault

[1]Paraphrasing George E.P.Box: "All models are wrong, but some of them are useful" [6].

[2]A property of some non-standard logics. It implies that if there is a change in opinions, or if more is learned, it is possible to arrive at a conclusion that something that was thought as true before is not true anymore. Thus, any logic that must deals with practical subjects should model non-monotonicity [31].

diagnosis, object recognition, biomedical engineering, autonomous vehicles navigation, climatology, simulation, and target identification. Sentz's and Ferson excellent work [42], supplies 148 references for applications of the theory, classified by segment, such as cartography, classification, decision-making, specialist systems, fault detection, medical applications, sensor fusion, risk analysis and reliability, and robotics.

Among the technical characteristics which motivated us to choose the Dempster-Shafer Theory, we can list [42]:

- The relatively high degree of theoretical development.
- The close relations between it and Probability and Set Theories.
- The versatility in the representation and combination of different kinds of evidence obtained from multiple sources (including mixtures of objective and subjective evidence).