INVESTMENT IN DEVELOPING COUNTRIES: EXPLORATIONS IN CAPITAL FLOWS, PRODUCTIVITY AND MICROADJUSTMENT

by
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Abstract

The enhanced access of developing countries to the international financial market since the seventies has been characterized by boom-bust cycles of unfettered external borrowing followed by abrupt financial crises. The first chapter analyzes the macroeconomic effects of volatile capital flows to a developing country. The analysis shows that investment, consumption, and the current account deficit depend positively on the expected availability of external finance. If international investors may unexpectedly decide to reduce their exposure to financial assets issued by the country, the optimal cost of external borrowing should exceed the interest rate paid by domestic residents in the international financial market. In the absence of insurance markets for this type of risk, a tax on capital inflows can be optimal.

Recent endogenous growth models characterize a firm's technology as a commodity which is both partly excludable and associated with some production inputs, such as human capital and equipment. The second chapter explores the nature of the link between equipment investment

and technology at the plant level in a large sample of Colombian manufacturing establishments. The results support the endogenous growth model's notion that technology is associated with the production inputs. Larger plants that invest more in machinery and equipment and employ higher levels of human capital tend to be more efficient.

Models of investment with non-convex costs of adjustment predict that microeconomic time series of investment may be characterized by infrequent investment spurts and prolonged periods of little or no investment. In the third chapter I study the pattern of investment at the plant level in different categories of capital goods. As in the U.S., plant-level investment in Colombia is lumpy, and the probability of observing a large investment episode depends positively on the time elapsed since the latest large investment episode. As a contribution to the literature, I propose and implement two alternative econometric methods for the estimation of a simple model of irreversible investment. The results show that increases in the real exchange rate (pesos per dollar) have a consistently negative effect on investment, regardless of the type of capital good.

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Investment in Developing Countries:
Explorations in Capital Flows, Productivity and Microadjustment

Introduction

This dissertation deals with three questions: (i) Why do investment and international capital inflows move in the same direction? (ii) Does equipment investment boost productivity growth? and (iii) What are the determinants of manufacturing investment at the plant level?

The first question is inspired by the cyclic character of capital movements towards developing countries since international financial markets grew in volume of operations and sophistication in the late sixties. Many developing countries have experienced booms and busts in consumption and investment which coincided with booms and busts in their access to the international financial market. The first chapter of this dissertation examines that connection with a simple macroeconomic model. The main hypothesis advanced is that investment may fluctuate too much because agents of the borrowing country do not know their intertemporal budget constraint. If they incorrectly infer that large capital inflows today signal large capital inflows tomorrow, the economy will engage in a path of high expenditure and current account deficits every time that the international financial market is in a boom. As a result, investment (and consumption) will need to be cut if international investors unexpectedly decide to reduce their exposure to financial assets issued by the country.

If investment were associated with productivity growth, a drop in investment caused by an external financial shock would not only reduce the country's accumulation but also hamper its prospects for long term growth. In the second chapter, I empirically investigate the linkage between investment and productivity. Differently from most existing studies, I employ a microeconomic data set of Colombian manufacturing plants during the period 1974-91. The results of the analysis show a strong relationship between the levels of investment and productivity, but no clear relationship between the level of investment and the rate of growth in productivity.

While the first question refers to the broad macroeconomic context in which investment takes place and the second examines the implications of investment for growth, the third one focuses on the determinants of investment at the microeconomic level. The third chapter studies investment at the plant level in Colombia. The analytical framework is given by the recent theories of irreversible investment, and the empirical work suggests a simple econometric implementation of such theories. The results show that increases in the real exchange rate (pesos per dollar) have a consistently negative effect on investment at the micro level.

Chapter One

Volatile Capital Flows to Developing Countries: Where is the Ceiling?

Summary

The enhanced access of developing countries to the international financial market since the seventies has been characterized by boom-bust cycles of unfettered external borrowing followed by abrupt financial crises. This chapter analyzes the macroeconomic effects of volatile capital flows to a developing country. The analysis shows that investment, consumption, and the current account deficit depend positively on the expected availability of external finance. If there is a risk that international investors unexpectedly decide to reduce their exposure to financial assets issued by the country, the optimal cost of external borrowing exceeds the interest rate paid by domestic residents in the international financial market. In the absence of insurance markets for this type of risk, a tax on capital inflows can be second best optimal.

Volatile Capital Flows to Developing Countries: Where is the Ceiling?

"There seemed to be no limit on our supply of credit, no concern about our ability to service it. We had high growth, a decade of political stability, and huge oil reserves. It seemed that nothing could go wrong." - A Mexican central bank official on the years that preceded the 1982 debt crisis. (Lissakers, 1991, p. 49)

1. Introduction

The dramatic expansion of international financial markets since the seventies has allowed an increasing number of developing countries to finance investment and consumption expenditures by selling debt instruments to private international investors. However, the access of these developing countries to international financial markets has been far from smooth. Boom-bust cycles of unfettered external borrowing followed by abrupt financial crises have been all too common. The most notorious of these cycles are the banking lending boom preceding the international debt crisis of 1982 and the large expansion of portfolio investment leading to the Mexican balance of payments crisis of December 1994.[1] In both cases the busts that followed the booms were abrupt, mostly unexpected, and they affected many countries simultaneously.

[1] Many recent papers address the surge of capital inflows to developing countries in the early nineties. See e.g. Devlin, Ffrench-Davis, and Griffith-Jones (1995); Gavin, Hausman, and Leiderman (1996); Calvo, Leiderman, and Reinhart (1996); Corbo and Hernandez (1996); Helleiner (1996); and Fernandez-Ariaz and Montiel (1996).

In a financially open economy, consumption and investment decisions are not constrained by the current level of the country's GDP. The access to the international financial market makes it possible, and indeed desirable, to finance consumption and investment expenditures with external borrowing. The relevant macroeconomic budget constraint becomes intertemporal in nature, giving the country flexibility to spend in excess of its GDP in some years.

There are several theories of the determinants of countries intertemporal budget constraints. They all coincide in a fundamental point: that the level of the country's external debt might be constrained by a credit ceiling imposed by international investors. As we shall see in Section 2, solvency and sovereign risk considerations are two major reasons why international investors might be reluctant to increase their exposure to assets issued by a given country, but they are not the only ones. For developing countries, exogenous events, such as a hike in the U.S. interest rates, can also have an impact in their ability to borrow in international financial markets.

The main hypothesis advanced in this essay is that the external financial crises experienced by many developing countries in 1982 and 1994 are explained by the generalized lack of knowledge about the location of their respective credit ceilings. As the above citation suggests, policy makers of developing countries experiencing a boom in their access to the international financial market seem to have overestimated their future access to external borrowing. In other words, they seem to have ignored the possibility of hitting a credit ceiling that would imply the need for an abrupt and painful adjustment of the current account.

To support this argument, I develop a simple infinite horizon, representative agent macroeconomic model in which the stock of the external debt cannot exceed some credit ceiling.

The main result of the analysis is that under the assumption that consumers and firms of the borrowing country know the location of the credit ceiling, an abrupt adjustment of the current account deficit is inconsistent with economic optimality. The optimal adjustment of the current account deficit is smooth and starts well in advance the time when the external debt reaches its credit ceiling.

In order to analyze the effect of not knowing the location of the credit ceiling, I add to the basic model a "market access" shock which represents the risk that international investors may abruptly refuse to increase their exposure to the country’s liabilities. The optimal response of consumers and firms to the presence of uncertainty about the country’s access to the international financial market is to moderate the paths of consumption and investment expenditures. As a result the current account does not increase as fast, and the potential pain of having to adjust the current account precipitously can be significantly reduced.

The rest of the essay is organized as follows. In Section 2, I briefly review theoretical and empirical arguments that support the macroeconomic relevance of credit ceilings for developing countries. In Section 3, I present the basic model, which I extend to the case of uncertainty in Section 4. Some of the formal arguments are relegated to the Appendix. In Section 5, I discuss some policy implications of the analysis.

2. The Macroeconomic Relevance of Credit Ceilings for Developing Countries

A country's credit ceiling is the maximum exposure desired by international investors—be they commercial banks, institutional fund mangers, or individuals—to liabilities issued by the country's economic agents. Several theoretical and empirical arguments were advanced to explain the existence of credit ceilings in international finance. We can broadly classify them into those that depend on factors specific to the borrowing country and those that depend on factors beyond the control of the borrowing country. The main arguments of the first group are solvency and sovereign risk.

A country is solvent when it has the physical ability to repay its external debt in the long run. According to this argument, international investors would stop buying assets issued by the country when its external debt becomes so high that it cannot be materially serviced. Sachs (1984) defines insolvency as a situation in which debt service exceeds the share of GDP that is not used for gross investment. In such situation the external debt cannot be serviced even if consumption is zero. Similarly, Blanchard and Fischer (1989) define insolvency as a situation in which debt service can only be financed by issuing new liabilities (a Ponzi game).

The difficulty with the above definitions of insolvency is that they assume that the borrowing country's consumption can be squeezed as much as necessary in order to comply with external financial obligations. But sovereign debt contracts are not enforceable by law. A sovereign government may conclude that the sacrifice in consumption required to service the external debt exceeds the costs of breaching its debt contracts.

The reason why sovereign countries honour their external debts does not rest on the enforceability of international debt contracts. According to the theory of sovereign debt, the critical factor is the costs of default. As long as those costs exceed the costs of servicing the external debt, the borrowing country will comply with its obligations. Acknowledging this, international investors will be willing to purchase debt instruments issued by the country. Much discussion has taken place in this literature about the nature of the costs of default[2], but the general conclusion is that they are independent of the level of debt. In contrast, the costs of servicing the debt obviously increase with debt. As a result, there should exist a sufficiently high level of debt at which default is the least costly option for the borrowing country. Taking this possibility into account, international lenders will be reluctant to increase their exposure to the country's liabilities beyond some point.

Solvency and sovereign risk imply that the supply schedule of external finance to each individual country has a point where it becomes vertical. The reason why booms may be followed by busts is that the country's demand for external finance shifts fast to the right until it hits the point when the supply schedule becomes vertical. The main policy implications for the borrowing country are to moderate the growth of public and private consumption expenditures to avoid sharp increases in imports and to maintain the real exchange rate at a level that does not discourage the production of tradeables. Such policies moderate the speed at which the demand for external financing shifts to the right, and they may avoid a crash as the vertical section of the supply schedule is reached.

[2] See Eaton and Fernandez (1995) for a recent review.

Although theories of boom-bust cycles based on fast shifts to the right of individual country demands for international finance are plausible, they leave important questions unanswered: Why did so many different countries crash against the vertical section of their respective supply schedule simultaneously? Moreover, given the adverse economic consequences of those crashes, why did they crash at all? After all, they could have chosen to moderate their demand for international finance before the crash.

The second group of arguments to explain the existence of credit ceilings relies on factors beyond the control of the borrowing country. They take into account the supply side of the market as well. In a pioneering work, Calvo, Leiderman, and Reinhart (1993) found that capital movements towards developing countries depend significantly on factors beyond their control, such as U.S. interest rates. This result has been confirmed by many other researchers.[3] The main implication of this view is that a crisis may occur not only because of a rapid shift to the right of the county's demand for external finance but also because of an abrupt shift to the left of the vertical section of the supply schedule.

A theoretical rationalization of the adverse effect of a hike in the international interest rate on the access of developing countries to the international financial market was formulated by Stiglitz and Weiss (1981) fifteen years ago. They have shown that under conditions of asymmetric information, increases in the interest rate beyond some point may increase the riskiness of loans, reducing their expected return. As a result, it may be optimal for lenders to limit the interest rate they charge and set a ceiling to the amount they lend.

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[3] See Frankel and Okongwu (1996) for a brief review of this literature.

Stiglitz and Weiss also analyze the case in which lenders cannot identify the riskiness of individual borrowers. The most they can do is identify the average riskiness of different groups of borrowers. In that case, the interest rate charged to each group will differ, and the riskier groups can be altogether excluded from the credit market when the cost of funds rises. This characterization is particularly relevant for developing countries. It is quite likely that international investors classify then into a few broad groups according to their perceived riskiness.[4] Of these groups, the ones perceived as the most risky are likely to suffer the greatest variability in their access to the international financial market.

Including the supply side of the international financial market can account for the simultaneity of busts in different countries. Moreover, if market-wide shocks, such as a hike in the U.S. interest rate are unexpected, many individual borrowing countries can hit the vertical section of their respective supply schedules abruptly.

Regardless of the precise reason why so many developing countries have experienced abrupt crises in their access to the international financial market, all the explanations reviewed coincide in depicting the supply of external finance as vertical at some point. This credit ceiling may be determined by the country’s solvency, the limited ability of lenders to punish the borrower in case of default, or conditions in the U.S. money market. Whatever the reason, the implication for the borrowing country is the same: that the credit ceiling should be incorporated into the analysis and formulation of the country’s macroeconomic policies.

[4] During the boom of commercial bank lending to developing countries in the seventies, country spreads seems to have been determined in that way. See Devlin (1989, pp. 102-016).

The main obstacle to including the credit ceiling explicitly into the analysis is that it is an unobservable variable. The institutional setting in which international financial transactions take place is characterized by a myriad of banks, currency traders, institutional investors, and multinational firms pursuing disparate objectives and assessing risks in dissimilar ways. Each individual international investor is likely to set individual limits of desired exposure to debt issued by different countries. The credit ceiling of an individual country results from the aggregation of limits of desired exposure of individual international investors. Although each individual investor knows its own exposure limit to a given country, the individual limits of other market participants are unknown. As a result, neither individual investors nor the borrowing country are likely to know the level of the credit ceiling.

To overcome this problem, I treat the credit ceiling as a parameter that represents the expectations of the country's agents as to the conditions of the international financial market. More precisely, I define the credit ceiling as the maximum level of exposure to the country's debt desired by international investors, *as perceived by the country's own consumers and firms*. For simplicity, I assume that consumers and firms believe that international investors have the means to extract the contractual payments on whatever level of debt they choose to issue. Therefore, default is not perceived as a feasible policy option.

In this essay I do not discuss how expectations about the country's access to the international financial market are formed. Presumably, the current state of the market conveys some useful information: if consumers and firms observe that their country has received capital inflows over the last few years, and that the same is true for other developing countries of similar characteristics or in the same region, they might infer that this trend might continue for some time. A more pedestrian reason is that the country's decision makers are likely to read the leading international financial newspapers. Regardless of how the country's agents infer the location of the credit ceiling, the main hypothesis advanced in this essay is that such information is important to understand the country's macroeconomic performance.

3. The Basic Model

The economy is populated by *n* identical, infinitely-lived families who, in each period, consume a composite good and supply one unit of labour. The preferences of each family for consumption over time are represented by

$$U = \int_0^\infty e^{-\theta t} \frac{c_t^{1-\gamma}}{1-\gamma} \, dt \;, \quad \gamma > 0 \;, \quad \gamma \neq 1 \;, \tag{1}$$

where θ is the subjective rate of time preference and γ is the inverse of the intertemporal elasticity of substitution. The consumption good c_t is expressed in per capita terms.

The composite good is produced by a number of identical firms with a constant returns to scale technology that utilizes labour and capital as inputs. This technology can be represented by a strictly increasing and strictly concave function mapping capital per capita on production per capita, i.e. $y_t = f(k_t)$. I assume that $f(k_t)$ also satisfies $f'(0) = \infty$ and $f'(\infty) = 0$. The composite good can be used both for consumption and as capital. Capital accumulates according

to

$$\frac{dk}{dt} = i_t - \delta k_t , \qquad k_0 > 0 \quad given , \tag{2}$$

where i_t is gross investment per capita and δ is the rate of depreciation. Gross investment entails adjustment costs, represented by the quadratic function $g(i_t) = (a/2)(i_t)^2$, $a > 0$. These costs can be interpreted either as costs of installing new capital goods, additional production costs incurred when output will be used as capital, or costs to train the workers that will operate the new capital equipment. Letting m_t denote net imports per capita, the resource constraint of this economy is given by

$$c_t = f(k_t) + m_t - i_t - \frac{a}{2} i_t^2 . \tag{3}$$

I assume that the residents of the country can sell a debt instrument in the international financial market. This instrument pays an interest rate r defined as the sum of a reference market interest rate and a fixed spread that reflects the country's risk category.[5] I further assume that the debt instrument does not mature and that all the residents of the country are able to sell this debt instrument at the same terms (that is, they pay the same spread over the reference interest rate). Finally, the residents of the country can also purchase financial assets issued by other countries. For simplicity I assume that foreign assets do not mature and pay the

[5] This pricing scheme is representative of a large number of international debt contracts, such as bank loans (in which case the reference rate is the LIBOR or the U.S. prime rate) and dollar-denominated bonds (in which case the reference rate is usually the interest rate on bonds of the same maturity issued by the U.S. government).

same interest rate as the domestic asset.[6]

I assume that international investors do not want to hold more than a certain amount of debt issued by the country. As discussed in the introduction, a major problem for modelling the credit ceiling is that it is likely to be unknown, not just for the borrowing country but also for individual international investors. To overcome this problem, I simply define the credit ceiling as a parameter that represents the maximum level of exposure to the country's debt desired by international investors, *as perceived by the country's own consumers and firms*.

The supply of credit to the country is, then, represented by an inverted L-shaped supply schedule: External financing is available at a fixed cost as long as debt does not exceed the credit ceiling. A possible objection to this characterization is the presumption that the spread should rise with the level of debt, reflecting an increasing risk of default (as in Eaton and Gersovitz, 1981, and Kletzer, 1984). However, the experience of the seventies (when bank loans played a dominant role in developing countries' international borrowing) suggests that a flat supply schedule is a reasonable approximation. Özler (1992) has shown that the spread on loans to developing countries between 1968 and 1981, a period the external debt of developing countries was rising at a fast rate, was actually inversely related to the number of loans previously granted to the country. This implies an *inverse* relationship between spread and debt.

The experience of the nineties also suggests that a flat supply schedule is a reasonable working hypothesis. The spread on Mexican 3-month T-bills (Tesobonos) declined from 7% in December of 1989 to 2% in December of 1993. During 1994 it shot up shortly to 4% after the

[6] This assumption is justified if country spreads correctly reflect the risk of assets issued by different countries. In that case, the risk-adjusted interest rate will be internationally equalized. If asset holders are also risk-neutral, they will only care about the risk-adjusted rate.

assassination of the PRI presidential candidate, but then declined to less than 2% until the second week of December. Only *after* the balance of payments crisis erupted did the spread climb to close to 20% in January of 1995 (see Sachs, Tornell, and Velasco, 1996a). In sum, the experience of both the seventies and the nineties suggests that markets have failed to convey the information that higher levels of external debt are associated with higher risks of default—until crises occurred.

Denoting debt and the credit ceiling in per capita terms, respectively, by b_t and l, the evolution of the country's balance of payments is given by

$$\frac{db}{dt} = rb_t + m_t, \qquad b_0 \ \ given, \qquad b_t \leq l, \tag{4}$$

where the left hand side is the capital account surplus and the right hand side is the current account deficit.[7]

The problem of this economy is to decide at each point in time how much to consume, how much to invest, and how much to rely on external financing to achieve the first two purposes. To answer these questions assume the existence of a benevolent social planner wishing to maximize the welfare of the representative family. The social planner's decision problem consists in the maximization of (1) subject to (2), (3), and (4). The current-value

[7] For simplicity, the interest rate and the credit ceiling are assumed to be constant. The credit ceiling could be represented more generally as a function $l(r,z)$, where r is the interest rate and z is a vector of exogenous variables that consumers and firms use to forecast l. As long as z does not contain endogenous variables, there is no loss of generality in representing the credit ceiling as a constant.

Lagrangian of the problem is[8]

$$L = \frac{c_t^{1-\gamma}}{1-\gamma} + \lambda_t [i_t - \delta k_t] + \mu_t [rb_t + c_t + i_t + \frac{a}{2} i_t^2 - f(k_t)] + \eta_t [l - b_t] ,$$

where the costate variables λ_t and μ_t and the Lagrange multiplier η_t represent, respectively, the shadow values of capital, external debt, and the foreign borrowing constraint. Equation (2) plus the following conditions are necessary and sufficient for a maximum:

$$c_t^{-\gamma} = -\mu_t \tag{5}$$

$$\lambda_t = -\mu_t [1 + a i_t] \tag{6}$$

$$\frac{db}{dt} = rb_t + c_t + i_t + \frac{a}{2} i_t^2 - f(k_t) , \qquad b_0 \quad given \tag{7}$$

$$\frac{d\lambda}{dt} = \theta \lambda_t + \delta \lambda_t + \mu_t f'(k_t) \tag{8}$$

$$\frac{d\mu}{dt} = \theta \mu_t - r \mu_t + \eta_t \tag{9}$$

$$\lim_{t \to \infty} e^{-\theta t} \lambda_t k_t = 0 \tag{10}$$

$$\lim_{t \to \infty} e^{-\theta t} \mu_t b_t = 0 \tag{11}$$

---

[8] The first three terms of the Lagrangian correspond to the standard Hamiltonian. The last term adjoins the constraint to the level of the external debt (see Chiang, 1992).

$$l - b_t \geq 0 \qquad \eta_t \geq 0 \qquad \eta_t [l - b_t] = 0 \tag{12}$$

$$\mu_\tau^+ - \mu_\tau^- \leq 0 \tag{13}$$

Notice that the shadow value of external debt is the negative of the marginal utility of consumption (Equation 5). The economy can be either unconstrained (when $b_t < l$) or constrained (when $b_t = l$). While the economy is unconstrained, the model reduces to a standard neoclassical model of an open economy; while the economy is constrained, the model reduces to a standard neoclassical model of a closed economy. At the points in time when the economy switches from unconstrained to constrained (or vice versa), the shadow value of the external debt may shift down discretely (Equation 13). The next section briefly analyzes the benchmark case of an economy whose external debt is always below the credit ceiling.

3.1. The Unconstrained Economy

Proposition 1: In the unconstrained economy investment and consumption are chosen independently of each other. The investment path is chosen to maximize the country's wealth; the consumption path is chosen to maximize utility subject to the maximized wealth.

Proof: See Appendix.

According to Proposition 1, unrestricted availability of external financing helps the country to achieve its maximum possible levels of wealth and welfare. The following proposition characterizes the optimal solution.

Proposition 2: If $\theta \geq r$, the unconstrained economy converges to a steady state. If the capital stock is initially below its steady state level, the transitional dynamics are characterized by a monotonically increasing external debt.

While the external debt increases, the current account is in deficit; as debt converges to its steady state level, the current account deficit converges to zero. The current account deficit is the sum of interest payments on debt rb_t and the trade deficit, defined as the excess of domestic absorption $c_t + i_t + (a/2)(i_t)^2$ over production $f(k_t)$. Since interest payments increase with debt, the gradual reduction of the current account deficit requires a continuous improvement of the trade balance. In the steady state the trade balance should generate a surplus equivalent to the steady state interest payments on the external debt.

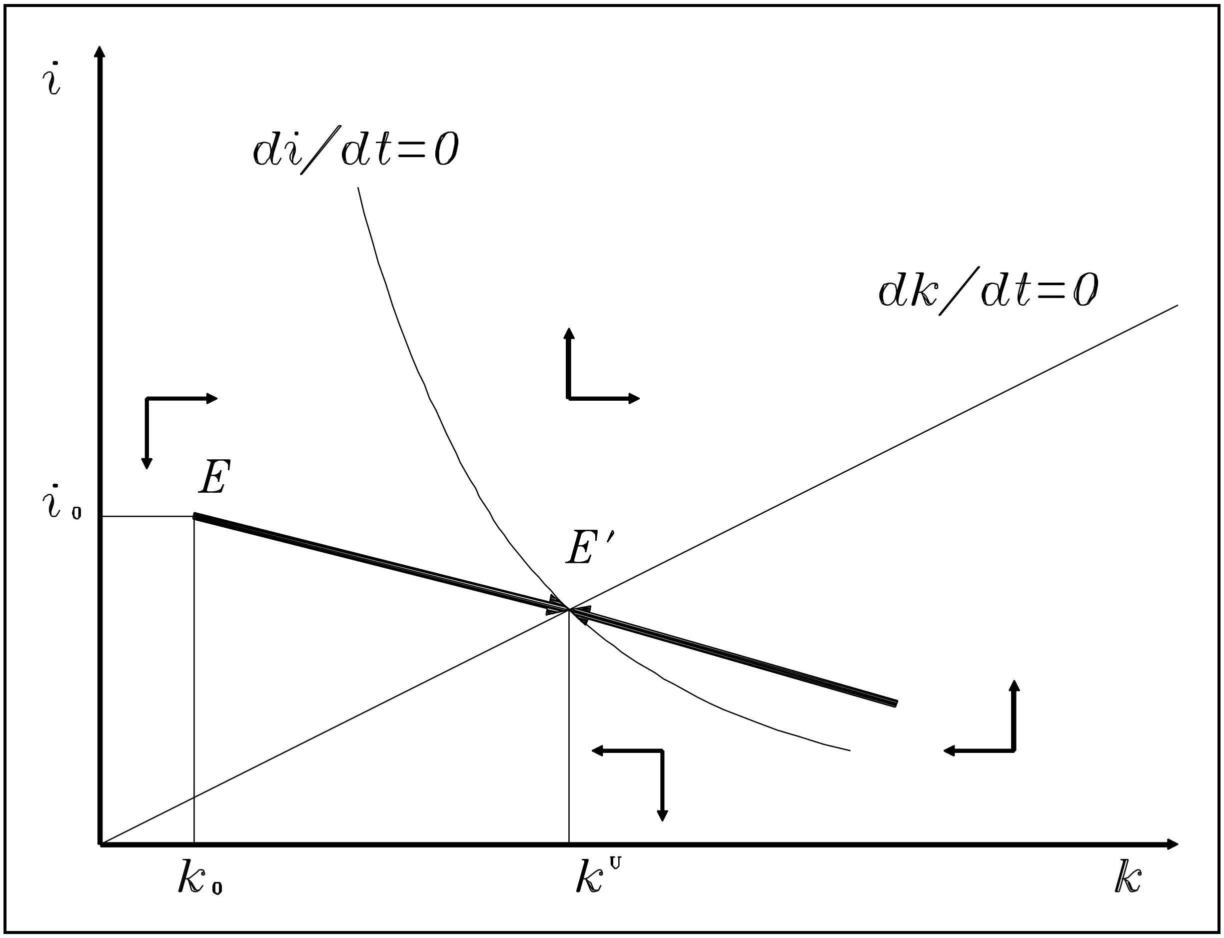
For the trade balance to improve over time, production should increase more than domestic absorption. Differentiating λ_t with respect to time in (6), substituting $d\mu/dt$ from (9), $d\lambda/dt$ from (8), λ_t from (6), and collecting terms, the law of motion of investment can be expressed as

$$\frac{di}{dt} = -\frac{1}{a}\left[f'(k_t) - (\delta + r)(1 + a i_t)\right]. \tag{14}$$

To study qualitatively the paths of investment and capital, consider the phase diagram of the system formed by Equations (2) and (14) in Figure 1.

The system is saddle point stable. If the initial capital stock k_0 is below its steady state level k^U, capital increases and investment declines along the optimal path *EE'*.

Figure 1: Unconstrained Economy - Phase Diagram of Investment and Capital



Letting q_t=1+a_t denote the relative shadow price of capital in terms of consumption, Equation (14) can also be expressed as the familiar neoclassical equality between the marginal product of capital and the rental price of capital:

$$f^{\prime}(k_t) \;=\; \left[\delta + r - \frac{1}{q_t}\frac{dq}{dt}\right] q_t . \qquad (15)$$

Given the definition of q_t and the fact that investment declines along the optimal path, it follows that q_t decreases over time until the steady state. As a result, the marginal productivity of capital exceeds the sum of the depreciation and the international interest rate times the price of capital during the transition to the steady state. In the steady state, capital satisfies

$$f'(k^U) \;=\; (\delta + r)(1 + a\delta k^U) \;=\; (\delta + r)\, q^U . \tag{16}$$

Differentiating μ_t with respect to time in (5), substituting $d\mu/dt$ from (9), μ_t from (5), and collecting terms, the path of consumption is

$$\frac{dc}{dt} \;=\; -\left[\frac{\theta - r}{\gamma}\right] c_t , \tag{17}$$

which integrates to

$$c_t \;=\; c_0 \, \exp\left[-\left(\frac{\theta - r}{\gamma}\right) t \right]. \tag{18}$$

In order to obtain the initial level of consumption c_0, substitute (18) into (7) and integrate between 0 and ∞ under the conditions $\theta \geq r$ and $b_t \leq l$:

$$c_0 \;=\; \left[r + \frac{\theta - r}{\gamma} \right] \left[\int_0^{\infty} e^{-rt} \left[f(k_t) - i_t - \frac{a}{2} i_t^2 \right] dt \;-\; b_0 \right]. \tag{19}$$

Although the economy never reaches its credit ceiling in the optimal solution, the condition $b_t \leq l$ is necessary to rule out an explosive path of consumption and debt. The no-Ponzi-game condition in Blanchard and Fischer (1989, p. 49) is a weaker form of this restriction. Similarly, the condition $\theta \geq r$ is sufficient to rule out an infinite accumulation of foreign assets by the country's residents. A weaker condition is $r + (\theta - r)/\gamma > 0$ (see Cha, 1992, p. 25).

As is clear from Equation (17), consumption declines over time if consumers are impatient ($\theta > r$) or remains forever equal to c_0 if $\theta = r$. As a result of this and the previous characterization of the optimal investment path, domestic absorption clearly drops over time. Since production increases as capital accumulates, the trade balance unambiguously improves over time. In the steady state the trade balance has a surplus and the current account is in equilibrium. The steady state level of debt of the unconstrained economy is:

$$b^U = \frac{1}{r}\left[f(k^U) - c^U - \delta k^U - \frac{a}{2}(\delta k^U)^2\right], \tag{20}$$

where steady state consumption c^U is either c_0 or 0.

3.2. The Constrained Economy

The analysis of the previous section is valid as long as the credit ceiling is larger than b^U. In a world of symmetric information and perfect enforceability of debt contracts, the only consideration of international investors is that the country remains solvent. Setting the credit ceiling at

$$l = \frac{1}{r}\left[f(k^U) - \delta k^U - \frac{a}{2}(\delta k^U)^2\right]$$

is enough for this purpose. Comparing this expression with Equation (20), it is clear that l is greater than or equal to b^U. But in a world of asymmetric information and imperfect enforceability of international debt contracts, international investors may not want to hold more than a fraction of b^U in debt instruments issued by the country. In this section I analyze an economy which is initially unconstrained but faces a credit ceiling that is smaller than the steady state level of debt of the unconstrained economy (i.e. $b_0 < l < b^U$). Throughout the analysis I maintain the assumption that $\theta \geq r$.

As the economy starts out unconstrained, the laws of motion of investment and consumption derived in the previous section (Equations 14 and 17) remain initially valid. We know by Proposition 2 that the optimal path of debt of an unconstrained economy is monotonically increasing until the steady state. This path is now unfeasible because debt cannot grow beyond some $l < b^U$. A first question that arises is whether the external debt will necessarily reach the credit ceiling. By selecting lower initial values, consumption and investment can still satisfy Equations (14) and (17) in such a way that debt is always below l. For example, assume that investment follows the same path as in the unconstrained economy, so that the paths of capital and production are the same. Recalling that in the unconstrained economy the current account deficit decreases over time, it is clear that a lower initial level of consumption will make the current account deficit decrease faster and the external debt increase more slowly. A sufficiently low initial level of consumption can make the current account deficit equal to zero at a level of debt less than the credit ceiling. But as the current account keeps improving, debt will start to decrease and eventually foreign assets will accumulate without bound, violating the transversality condition (11). Therefore, the external debt must reach the credit ceiling at some finite point in time.

A second question is what happens once the external debt reaches the credit ceiling. Will the economy return to the unconstrained regime? The answer is no. To go back to the unconstrained regime, the optimal or desired level of debt would need to decrease below l at some point in time. We know by Proposition 2 that this is not the case. Consumers and firms would like to issue more—not less—debt, but they cannot do it when the credit ceiling is binding. In synthesis, when the credit ceiling is greater than the initial level of debt but less than b^U, the economy will be unconstrained until the time when the external debt reaches the credit ceiling. Between that time and the steady state, the economy will be constrained.

The laws of motion of investment and consumption during the constrained regime can be derived, as before, from Equations (5), (6), (8), and (9), the only difference being that η_t is now non-negative (see 12). These are

$$\frac{di}{dt} = -\frac{1}{a}\left[f'(k_t) - (\delta + \rho_t)(1 + ai_t)\right] \tag{21}$$

and

$$\frac{dc}{dt} = -\left[\frac{\theta - \rho_t}{\gamma}\right]c_t, \tag{22}$$

where $\rho_t = r - \eta_t/\mu_t$ represents the domestic interest rate.

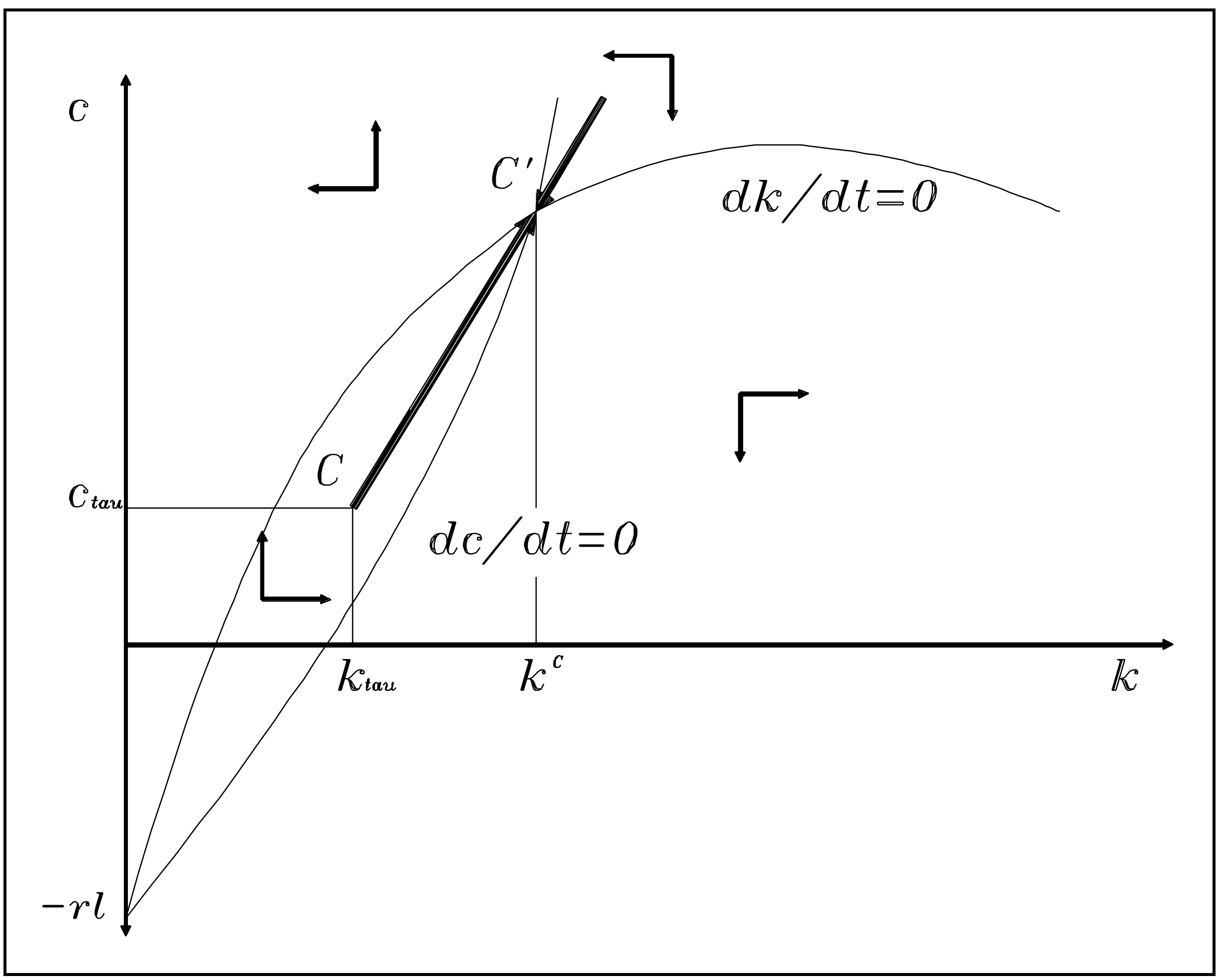
Recalling that μ_t is negative (see Equation 5), it is clear that the domestic interest rate exceeds the international interest rate during the constrained regime. When the country has no access to the international financial market, investment has to be financed from domestic savings only. As a result, the interest rate has to be high enough to induce consumers to save. In the constrained regime investment and consumption decisions are no longer independent of each other; they must be chosen in such a way that the country's current account remains in equilibrium. The domestic interest rate plays the same role in coordinating investment and consumption decisions as in a closed economy.

Proposition 3: The laws of motion for investment and consumption during the constrained regime are identical to those of a closed economy.

Proof: See Appendix.

In order to study qualitatively the paths of consumption and capital during the constrained regime, consider the phase diagram of Figure 2. The stable branch *CC'* is upward-sloping, so

Figure 2: Constrained Economy - Phase Diagram of Consumption and Capital



consumption increases as capital accumulates. As a result, the domestic interest rate ρ_t declines monotonically along the optimal path, converging to θ in the steady state (see Equation 22). Substituting the steady state level of ρ_t into (21) and setting di/dt=0, we obtain the steady state level of capital of the constrained economy:

$$f'(k^C) = (\delta + \theta)(1 + a\delta k^C). \tag{23}$$

If $\theta = r$, the constrained economy reaches the same steady state levels of capital and production as the unconstrained economy: $k^C = k^U$ (see Equation 16). The steady state level of consumption, however, is higher in the constrained economy, because a smaller part of output is devoted to interest payments on debt in the steady state. (See Equation 19, noting that the steady state level of debt is $l < b^U$.) If $\theta > r$, capital reaches a lower steady state level, but consumption is still higher than in the unconstrained economy because it converges to a positive level (see Figure 2).

We have so far characterized how the economy evolves during both the unconstrained and the constrained regimes. Our last questions are when and how does the economy enter the constrained regime. To investigate the first of these questions, define the maximum utility achieved during the unconstrained regime as

$$V^U(k_0, b_0; l) = \max \int_0^\tau e^{-\theta t} \frac{c_t^{1-\gamma}}{1-\gamma} dt \tag{24}$$

subject to (2), (3), (4), and the terminal condition $b_\tau = l$. Recalling the definition of the maximum utility achieved during the constrained regime (Equations A.9 to A.11 in the Appendix), the optimal time at which the economy enters the constrained regime can be obtained by solving