

GEOSPATIAL COMPUTATIONAL METHODS

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ALGORITHMS OF COMPUTATIONAL METHODS FOR
GEOMATICS, SURVEYING ENGINEERING,
GEOINFORMATICS, GEOSPATIAL INFORMATION
SCIENCE TECHNOLOGY (GIST), GEOGRAPHY,
GEOLOGY, AGRICULTURE, GEOINTELLIGENCE
APPLICATIONS

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Geospatial Computational Methods: Algorithms of Computational Methods for Geomatics, Surveying Engineering, Geoinformatics, Geospatial Information Science Technology (GIST), Geography, Geology, Agriculture, Geointelligence Applications

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To my wife, Nini
and to my children
Nafsika, Nikolaos and Despina

Preface

This book is for students and professionals involved in Geospatial Computations and related areas such as geoinformatics or Geospatial Information Science and Technology (GIS&T). Specifically, algorithms such as the least squares method to obtain ground coordinates of points with reliability analysis to construct the geometric framework of the geographical space are discussed. Such algorithms in the geospatial area are basic surveying methods using a total station, photogrammetry, digital terrain modeling, and GPS. It must be noted that the geometric framework of the geographical space and its reliability are of fundamental importance to building a Geographic Information System (GIS) to support applications and modeling environmental phenomena. The material covered is based on professional and academic experience in the USA and Greece. The presentation of the material is guided by the philosophy where scientific and technological aspects are considered mutually dependent on having the technology to implement the scientific analysis through software. This philosophy was developed by the first author during his academic career at Fresno State, USA (1980 – 1989) and continued at the University of Aegean Greece (1989 – 2015) with great success.

The idea is that the geospatially related material at the professional level must provide basic knowledge on understanding systems from the inside instead of the technician level, where operators treat systems as black boxes. It is essential to understand that most GIS&T operations are computerized and use artificial intelligence (AI), and the ultimate objective of this work is to help students and professionals *become more intelligent than machines*. It helps to obtain high-quality scientific and technological bases, which in turn enhance the ability to exploit and use most tools and functions of existing GIS&T systems and, therefore, to be highly competitive as a professional in the market.

The structure of this book includes simplified theory to derive necessary formulas translated into algorithms of corresponding software modules. People reading this book are educated about specialized mathematics and their implementation by an optimized algorithm and corresponding software to solve specific geospatial problems. Developed educational software by the first author accompanying this book accelerates the learning process and helps to understand and efficiently use related professional software. This book tries to resolve the myth of the "*unlimited smartness of the machine*" and provides scientific and technological support to the reader to achieve quality and excellence in professional development in this area.

It is evident today in the weakness of most Educational Institutes to attract people who combine scientific quality and technological advances (moving too fast) supporting the academic geospatial

processes. This fact creates an education gap, particularly in geospatial analysis programs. The industry, on the other hand, can attract such people for their business products and services. However, the industry holds a significant portion of the education and training in the geospatial area by exploiting the weaknesses of Education Institutes. There is no doubt about the quality of work the industry does in its educational programs, but the gap exists because it lacks academic freedom. This book provides the opportunity to close this gap since it involves professional and academic experience and maintains a didactics philosophy based on science and technology.

The structure of the book

This book is composed of ten chapters.

Chapter One covers the general concept of errors and the methods used for control and analysis based on probability theory and statistics.

Chapter Two deals with specialized numerical methods covering operations with matrix algebra, matrix inversion methods with emphasis on Cholesky's method, solution of linear equations, and solution of nonlinear equations with the Newton-Raphson method. Methods involving neural networks, genetic algorithms, and Cellular automata are also covered and analyzed.

Chapter Three starts with error propagation using both Taylor series analysis and Monte Carlo methods and then discusses the least squares of indirect observations, or observation method using linear and nonlinear equations.

Chapter Four discusses the least squares of observations only, the condition method, and the Generalized Least Squares adjustment.

Chapter Five deals with applications to map projections and using least squares to transform coordinates. Two-dimensional conformal, affine, bilinear, and projective transformations are covered. Three-dimensional conformal, or seven-parameter transformation and three-dimensional affine with 12 parameters are also covered. In addition, the polynomial best fit with the necessary parameters to cover radial lens distortion in photogrammetry is also presented. Furthermore, the celestial coordinate systems are presented, and some simple geodetic astronomical computations are formulated.

Chapter Six deals with applications of least squares to surveying networks using a total station. It provides plane network design and adjustment algorithms, a three-dimensional network of slope distance and vertical angle adjustment, and level network adjustment. It also covers applications of least squares in global positioning system (GPS) navigation solutions.

Chapter Seven deals with applications of least squares to photogrammetry. Resection intersection methods, image correlation to identify conjugate points in the common overlap of digital images taken from different exposure stations, epipolar geometry, and bundle methods are covered. In addition, photogrammetric applications with X-rays, drone photogrammetry, and augmented reality are formulated and presented. Photogrammetric methods, which use polynomial nonprojective geometry sensor models, are also formulated by least squares.

Chapter Eight applies interpolation methods to Digital Elevation Models (DEM) and applications in contour line calculation, perspective view visualization, derivation of shades, and water management computations.

Chapter Nine deals with how computer programming works using object-oriented programming with Visual Basic 6 and Python. Furthermore, it describes the computer structure and architecture and how it works with bits and bytes. Scripting applications of Python on open-source, object-oriented programming are presented. Therefore, sample scripts are presented on vector processing using the OGR library and raster processing using GDAL and Numeric libraries. It also introduces the reader to big data, remote access, cloud computing, the artificial intelligence (AI)

concept and methods, the dangers resulting from AI, and the ethics necessary to minimize such dangers.

Chapter 10 describes the associated with this book, educational software developed by the first author, and it is delivered together with this book.

Related Ethics

The authors desire to provide education through the present book and the knowledge acquired by reading it to be used constructively. For this reason, several foundation bases of education are listed below, which may contribute to this direction¹ (see also Chapter 9.8.10).

1. Education must be the *therapy of the human spirit* (human thought).
2. Education must be in *harmony with Nature*.
3. Education must cultivate a *sense of the beautiful*.
4. Education must aim at the virtuous with *internal and external balance*².
5. *Internal balance* is obtained by continuously maintaining sufficient *logic (reason)* to manage the *non-cultivated* parts of the thought *desire* and *anger* for constructive performance.
6. *External balance* is obtained by a continuous effort so that thoughts and actions are not *defective* or *excessive* but maintained within an intermediate space between defect and excess called the *mids-pace of virtue*³.
7. Human thought is the highest power in the Universe because it is the only means to understand its existence and do exploration. Without it, the Universe has no meaning of existence.
8. The power of human thought increases as the effort for internal balance increases. Therefore, the degree of maintaining sufficient logic determines the person's power.
9. The power of human thought increases as the effort for external balance increases.
10. The virtuous is the only person being educated and having enough power to work on the quality of life of a society and support exploration. They support justice, considering guilty whatever disturbs the internal and external balance. They also support democratic procedures as those defining the limits of the mid-space of virtue with a broader consensus.
11. Justice comprises all virtues, and Themis considers guilty whoever disturbs the scale's balance and punishes by the sword.

Notice: The regular performance of Nature objects requires obedience to Nature's rules and laws. Nature Laws are mandatory, and Nature rules have tolerance limits and exceptions. The most important rule is the one of the equilibrium or balance and contains all the rules. The Aristotelian mid-space of virtue belongs to *Nature's rules* and has universal validity. Such rules imply the specification variance Nature follows to obtain diversity and evolution. Outside of the rule's tolerance limits, there is a malfunction in the performance of Nature objects with undesirable consequences. For example, the Earth's orbit around the Sun is the balance of an attractive force due to gravity and an opposite force due to the rotary orbit. Therefore, the Earth never follows the same orbit path. Therefore, there is a mid-space of orbits with limited bounds (an average with its

¹ Work published by the author: "Practical philosophy of thought and virtue", Universal Publishers, 2004. "Education and Neuron Network Based Systems" - The scientific bases for the educator, Monogram, LAP publishers, 2014.

²Plato "The republic"

³Aristotle "The Nikomachean Ethics"

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variance), where such orbits occur to have equilibrium. Otherwise, Earth collides with the Sun or gets lost in space if it gets off such bounds. This idea of the mid-space of virtue helps to define precisely the boundaries of wrong and right. Within such boundaries is defined as the freedom of the Earth's movement.

Similarly, heartbeats or blood pressure out of bounds is an illness. Therefore, human freedom exists only within the bounds of the mid-space of virtue. Outside these bounds, there is illness, deception, and slavery⁴.

⁴ Aristotle "The Nikomachean Ethics"

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CHAPTER ONE

Measurements and Errors Estimation and Accuracy Standards

1.1 General

Geospatial computational tools and techniques deal with data collection and data processing. Data collection is either by direct measurements, archived data, processing existing data, or combining all such collections and processing methods. Data collection and processing must comply with two crucial issues: the data values (ground coordinates) used in various applications and the reliability of such data values. Therefore, we discuss various methods of collecting data and associated algorithms to obtain, i.e., coordinates of points and reliability analysis. Most applications use coordinates of points to build the geometric framework of the geographical space and support activities to develop a Geographic Information System (GIS), which helps to manage applications and modeling of various phenomena.

The emphasis in this Chapter is on the *stochastic processes*. Historically, the Greek philosopher Aristotle used the word "stochastic" for such processes to express the mean value of a sample together with its variance.

... τοῦ μέσου ἂν εἴη στοχαστική

(the mid-space location is *stochastic*, "Aristotle: The Nicomachean Ethics, paragraphs B-6, B-7)."

On the other hand, stochastic processes assemble all rules governing real-world phenomena. All real-world objects have attributes with parameter values, which are developed within certain variance limits to ensure the necessary freedom for their performance, growth, and their evolution. For example, an object position, such as a point position near the Earth's surface, is represented by parameter values, as are the coordinates. As defined by specifications, coordinates are determined through a stochastic process to control their variance limits (performance or precision). At the same time, science and technology are continuously evolving into new ways to determine the coordinates of a point with specified accuracy standards.

1.2 Topics covered in this Chapter

Data collection is usually measured or estimated quantities of real-world parameters. Such measurements and estimations carry error components, which characterize their reliability. This Chapter aims to model such error components and study their behavior. To achieve this, we define a mathematical model as an effort to simulate real-world objects having two parts: the functional part involving the real-world parameters and the stochastic part involving the distribution of errors associated with these parameters. For example, we measure the three angles of a triangle, which are the real-world parameters, and then the functional part of the mathematical model relates these three angles in a way that their sum must be 180 degrees. The stochastic part of the mathematical model states that due to random errors in the measurements, this sum would not be 180 degrees, and it provides scientific ways to use these measurements to estimate values that fit precisely the mathematical model. It also estimates the reliability of such values.

Therefore, measurements and errors implement statistical methods to study the behavior of random errors and their impact on reliability analysis by estimating values for real-world parameters. In addition, some theoretical issues about probability distributions are covered regarding crucial formulation, algorithms, and corresponding software to obtain results. Consequently, the emphasis is on the geometric interpretation of error analysis in the X and Y coordinates with an error ellipse and its significance.

Statistical analysis, or a stochastic process, is the next topic covered in detail with sampling theory, estimation methods, confidence intervals, statistical testing, and empirical estimation methods. Furthermore, we also present Baye's theorem.

This Chapter concludes with Map accuracy positioning standards by presenting several authoritative standards.

1.3 The stochastic and the functional part of the mathematical model

The *population* of measurements of a quantity (for example, a distance length) may have a theoretically infinite number of values. That is to say, each time we measure this length, it is always possible to measure it more times. Consequently, according to *probability theory*, the *population* of measurements of this length is composed of an infinite number of such individual survey values. Each survey value is considered a *random value* because we do not know this value unless we perform the survey. Therefore, all surveyed values of the same length constitute the total *population* of measurements of this length. This population is assumed to have a *normal distribution* with *mean value* μ and *variance* σ^2 . These two parameters, μ and σ^2 , define the *stochastic part* of the *mathematical model*. We mention again that a mathematical model is composed of two parts of the *functional part* and the stochastic part. The functional part describes the mathematical *equations*

(*functions*) which relate individual model parameters. These equations (functions) help us measure some model parameters, i.e., angles, distances, phase, and time, to calculate others, i.e., coordinates.

Probability theory, as it was said, considers one measurement of a length as being a value of a *population* with an infinite number of values. Consequently, *statistics* is working out to determine the parameters of a stochastic model (μ, σ) from a subset of population values or a *sample*, and they use statistical tests to conclude whether or not these values calculated from the sample are consistent with those of the entire population. On the other hand, geospatial processing uses a finite number of measurements or samples to calculate the mathematical model parameters. Usually, they are the coordinates of points and their precision (μ, σ), so most methods to be developed here (i.e., least squares) are statistical.

Statistics work better when there are as many redundant measurements or degrees of freedom as possible. If, for example, we measure a length only once, then there is no redundancy or zero degrees of freedom; therefore, no statistical treatment is allowed. In another example, if a length is measured three times, then there are two redundant measurements, and therefore, there are two degrees of freedom, which allow statistical analysis to calculate and make estimates of the mean (μ) and the variance (σ^2). In Geospatial computations, parameters (μ), which are usually the coordinates, help to draw a map, and the variance (σ^2) helps to control the precision of a map compared to map accuracy standards.

1.3.1 Measurements and Errors

Many geospatial projects deal with determining several points called *control points* or *new points*. Usually, we create such points by conventional surveys, *GPS*, *photogrammetry*, *Lidar*, *UAS*, *Augmented Reality*, *IfSar*, or a combination of most. In any case, we perform several measurements using various instruments. The result of a measurement is a value representing the magnitude of a specific quantity. As such, because of random *conditions* and limitations of the personnel and equipment involved, it is not an exact value of the measured quantity, but it has an error. For example, if the exact or true value τ of a measured quantity is known, and the measured or observed value is l , then the difference between l and τ is the error ϵ as follows:

$$\epsilon = l - \tau \quad (1.1)$$

It is essential to understand that true values could exist, and if, for example, one takes the coordinates of two fixed points A(X_a, Y_a) and B(X_b, Y_b), then the true distance d_{AB} between these two points is given by the Pythagorean theorem:

$$d_{AB} = \sqrt{(X_b - X_a)^2 + (Y_b - Y_a)^2} \quad (1.2)$$