Remarks on The Classical Theory of Fields

REMARKS ON THE CLASSICAL THEORY OF FIELDS

CORRECTIONS AND SUPPLEMENTS TO THE CLASSICAL ELECTRODYNAMIC PART OF LANDAU AND LIFSHITZ'S TEXTBOOK

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Remarks on The Classical Theory of Fields: Corrections and Supplements to the Classical Electrodynamic Part of Landau and Lifshitz's Textbook

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Preface

This book undertakes the unusual task of correcting and proposing supplements to the Landau and Lifshitz's highly celebrated textbook titled: *The Classical Theory of Fields*[1] [1], which has been extraordinarily influential. Its first edition was published in 1939, and new editions in many languages continue to be published. Furthermore, it is still cited in scientific works several hundred times each year. (As a personal remark: I regard Landau and Lifshitz as my most important indirect physics teachers. Unfortunately, for well-known reasons, I'm unable to consult them about my book; however, I have a deep feeling that they would agree with its main points.)

At present, it is rare to correct a well-established and fundamental physical theory. Discussing the reasons for this state of affairs is beyond the scope of this book; nevertheless, the book criticizes [1], and in so doing, it relies on well-established physical principles and a coherent mathematical analysis. Dirac gave the following description of the merits of mathematics in the framework of physical theories:

> The need for putting the mathematics first comes from its more rigid nature. One can tinker with one's physical or philosophical ideas to adapt them to fit the mathematics. But the mathematics cannot be tinkered with. It is subject to completely rigid rules and is harshly restricted by strict logic [2].

Readers may wonder how, after so many years and such scrutiny, one can still find incorrect points in a celebrated textbook like [1]? Here is just one brief example readers may find convincing regarding the need for this book to address such points. It pertains to contradictory statements about a crucial element of theoretical

[1] This textbook is cited many times. Therefore, readers can easily remember the citation number [1], and there is no need to consult the reference.

physics that are made in textbooks written by eminent physicists. Both the Landau and Lifshitz textbook (see [1], p. 48) and the book by Feynman, Leighton, and Sands [3] claim that the electromagnetic four-potentials are components of a four-vector A_μ. Many other textbooks make the same statement. However, according to Weinberg's textbook,

> The fact that A^0 vanishes in all Lorentz frames shows vividly that A^μ cannot be a four-vector (see [4], p. 251).

The four-potential A_μ and its gauge transformations play a key role in the present structure of theoretical physics. Therefore, I'm quite sure that every interested physicist would like to understand whose opinion is right in terms of the contradictory views about the four-potential of electromagnetic fields. This book provides judicious resolutions to this dilemma, as well as to some other problematic issues.

The topics discussed in this book have general relevance. They pertain not only to the Landau and Lifshitz textbook [1] but also to the presently accepted form of electrodynamics derived from the least action principle. This indicates the significance of this book.

Many people have helped me in one way or another to accomplish some tasks related to my scientific work. In particular, I wish to thank my family members for their encouragement and support. I am also grateful for the help of the following people: Prof. S. Rosset, Prof. Z. Schuss, and Prof. D. Levin from the School of Mathematics, Tel-Aviv University; Prof. K. T. Hecht and Prof. J. Janecke from the University of Michigan, Ann Arbor; K. Hellreich, who lived in Ann Arbor in 1983; A. Ney from Paris and C. Botner, P. Einat, Y. Tal, Y. Lev, and M. Meiri from Israel. I owe special thanks to N. Zeldes and I. Kelson, my M.Sc. and Ph.D. instructors, who taught me specific physical topics and general issues related to conducting scientific research. Finally, I want to acknowledge correspondence with Dr. G. Bella of the School of Physics, Tel Aviv University.

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Contents

Chapter 1

Introduction

This book discusses the electromagnetic chapters of Landau and Lifshitz's textbook [1]. The corrections and the supplements are theoretically significant. However, the contents indicate that the general structure of the textbook [1] is unshaken. Therefore, the detailed corrections and supplements presented herein can be regarded as a source of some replacements and extensions of [1]. This book assumes that the reader has adequate knowledge of the electromagnetic chapters of [1]. For example, it discusses the retarded Lienard–Wiechert four-potentials and their fields, but it does not explain the concept of retardation.

This book adheres to the principle that Wigner described as "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" [5]. I'm quite sure that Landau and Lifshitz would agree with this principle. In particular, they would have rejected any physical theory based on an incoherent mathematical structure. Following this approach, this book attempts to examine the mathematical structure of the classical theory of electromagnetic (EM) fields and the relevant experimental data.

Arguments that depend on quantum theories of electrodynamics have supportive role in this book. This means that these arguments are not completely disregarded; however, it is explained in the appropriate places that these arguments provide support for some of the assertions of this book.

1.1 Notation and Units

This book uses several conventions, some of which differ from those of [1]. $\boldsymbol{B}$ denotes the magnetic field in a vacuum. Greek indices run from 0 to 3. (In quotations from [1], Greek indices replace Latin indices.) The system of units where $\hbar = c = 1$ is used. There are additional notations used herein: A power of length $[L^n]$ denotes the dimension of every term. The Lorentz metric is diagonal, and its entries are (1,-1,-1,-1). This metric is used in the 2005 edition of [1]. Standard relativistic notation is used. The term $\epsilon^{\alpha\beta\gamma\delta}$ is the completely antisymmetric unit tensor of the fourth rank, and $\epsilon^{0123} = +1$. The Dirac $\boldsymbol{\alpha}$, β, and γ^μ matrices take the form in [6].

Following [1], the variational principle plays a primary role in this book. A Lagrangian is used in the case of classical particles, whereas a Lagrangian density is used for EM and quantum particles. This is because the celebrated textbook on classical electrodynamics [1] uses hybrid expressions: A Lagrangian describes the motion of a classical pointlike charged particles, and a Lagrangian density describes the laws of EM fields.

Magnetic monopoles represent an important part of this book. In electrodynamics, the term "charge" denotes an electric charge. Therefore, the term "magnetic charge" may confuse readers. Following Dirac, the term "magnetic strength" or "monopole strength" denotes the magnetic analog of the electric charge [7]. Nevertheless, it should be noted that the word "strength" may also be used in other senses.

This book discusses corrections and supplements to [1]. The symbol (C) denotes chapters that contain corrections, the symbol (S) denotes chapters that contain supplements, and the symbol (CS) denotes chapters that contain both.

1.2 Acronyms

The acronyms listed below are used in the book. However, full terms are still used in some cases.

1. CI – Configuration Interaction
2. EPP – Elementary Pointlike Particle
3. MLE – Maxwell Lorentz Electrodynamics (see section 2.5)
4. NRCPH – Nonrelativistic Classical Physics
5. QCD – Quantum Chromodynamics
6. QED – Quantum Electrodynamics (as known in 2022)
7. QFT – Quantum Field Theory

8. QM – Quantum Mechanics
9. RCMT – Regular Charge–Monopole Theory (see section 6.3
10. RQM – Relativistic Quantum Mechanics
11. SI – Strong Interactions
12. SM – Standard Model of particle physics
13. SR – Special Relativity
14. VE – Variational Electrodynamics (see [1])
15. VMD – Vector Meson Dominance

Chapter 2

General Principles

Physics is a mature science, and its theories abide by multiple principles with proven validity. This chapter describes several of these principles, providing a basis for the analysis presented in this book. Landau and Lifshitz use special relativity (SR) as a cornerstone for electrodynamics [1]. The present book adopts this approach, using SR as a fundamental theory.

2.1 The Correspondence Principle

In a physical theory, it is important to adequately define its validity domain. For example, nonrelativistic classical physics (NRCPH) is a good theory for describing processes that take place in the macroscopic world, and the velocity of the particles is much less than the speed of light. Quantum mechanics (QM) is restricted to cases where relativistic effects can be ignored. Hence, the validity domain of NRCPH is a subset of the validity domain of QM. For this reason, NRCPH is regarded as a theory with lower rank compared with QM.

The correspondence principle says that an appropriate limit of quantities of a theory with higher rank should agree with corresponding quantities of a theory with lower rank. This means that QM must define the particle density, energy density, and momentum density of the quantum particle *and* that the classical limit of these quantities should agree with the corresponding quantities of NRCPH. The need to prove the correspondence between QM and NRCPH was recognized in the early days of QM when the Ehrenfest theorem was published (see [8], pp. 25–27, 137, 138). Pages 1-6 of Rohrlich's book [9] contain a good discussion of the concept of correspondence between physical theories.

QM does not aim to explain everything. For example, experiments show that the proton comprises quark–antiquark pairs of the u, d, s flavor [10, 11]. In principle, this kind of evidence should be explained by quantum field theory (QFT). Hence, the domain of validity of QM is a subset of the domain of validity of QFT. This means that analogous constraints apply to QFT: QFT must define the particle density, energy density, and momentum density of the quantum particle, *and* the appropriate limit of the QFT quantities should agree with the corresponding quantities of QM. This correspondence is clearly stated in Weinberg's QFT textbook (see [4], p. 49):

> First, some good news: quantum field theory is based on the same quantum mechanics that was invented by Schroedinger, Heisenberg, Pauli, Born, and others in 1925-26, and has been used ever since in atomic, molecular, nuclear, and condensed matter physics.

The concept of correspondence entails two kinds of relationships between the relevant theories, which are as follows:

R.1 The lower rank theory imposes constraints on the appropriate limit of quantities of the higher rank theory. One aspect of this requirement is that the higher rank theory must define the physical variables of the lower rank theory.

R.2 A physical law that does not vanish in the limit process of a higher rank theory also applies to its lower rank theory.

Although this book discusses classical electrodynamics, there are a few cases where it applies the law R.2 and uses arguments that are proved in the quantum domain.

2.2 Lagrangian Density

The general community recognizes that a field theory of an elementary particle is based on the minimal action S of a Lagrangian density with the general form of

$$\mathcal{L}(\psi, \psi_{,\mu}). \tag{2.1}$$

This principle is the cornerstone of [1]. Today, QFT textbooks also use this principle for an elementary quantum particle (see, e.g., [4],

p. 300). The equations of motion of the function ψ are the Euler–Lagrange equations that are derived from a variation of the action S of (2.1) with respect to this ψ:

$$\begin{aligned} 0 &= \delta S \\ &= \delta \int \mathcal{L}\, d^4x \\ &= \int \left[\frac{\partial \mathcal{L}}{\partial \psi} \delta\psi + \frac{\partial \mathcal{L}}{\partial \psi_{,\mu}} \delta(\psi_{,\mu}) \right] d^4x \\ &= \int \left[\frac{\partial \mathcal{L}}{\partial \psi} \delta\psi - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \psi_{,\mu}} \right) \delta\psi + \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \psi_{,\mu}} \delta\psi \right) \right] d^4x. \end{aligned} \quad (2.2)$$

The integral of the last term of (2.2) yields the values of $\frac{\partial \mathcal{L}}{\partial \psi_{,\mu}} \delta\psi$ at spatial infinity. It is assumed that at spatial infinity, $\psi = \psi_{,\mu} = \delta\psi = 0$. Therefore, the form of (2.1) proves that the last term of (2.2) can be removed. The minimal action entails that the variation δS is zero. Since the variation $\delta\psi$ is an arbitrary quantity, it emerges that the first and second terms on the right-hand side of the last line of (2.2) yield the partial differential equations called the Euler–Lagrange equations

$$\frac{\partial \mathcal{L}}{\partial \psi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \psi_{,\mu}} \right) = 0. \quad (2.3)$$

In the case of EM fields, this equation boils down to the inhomogeneous Maxwell equation (see, [1], p. 78).

In the units used herein, the action S is dimensionless. Therefore, the dimension of the Lagrangian density is $[L^{-4}]$. Moreover, if the Lagrangian density is a Lorentz scalar, then the Euler–Lagrange equations take the required Lorentz invariant form (see, e.g., [4], p. 300). This is an important feature of applying the Lagrangian density as the basis for the theory: If it is a Lorentz scalar with a dimension of $[L^{-4}]$, then the theory abides by special relativity (SR).

Another important property of the Lagrangian density is its mathematically real form. This property is obvious for the EM fields, which are mathematically real quantities. However, it is interesting to note that this point is also true for mathematically complex quantum functions. Indeed, the integration factor d^4x is a mathematically real Lorentz scalar. Thus, the action S of such a Lagrangian density is a mathematically real Lorentz scalar. Such an action is used for the undulating factor of the particle's function of QM

$$\Phi = e^{iS} \quad (2.4)$$

(see, e.g., [12], pp. 127, 128; [13], pp. 19, 20). Evidently, the power series expansion of the exponent of (2.4) proves that a coherent action S should be a mathematically real Lorentz scalar. Furthermore, this form of the action is used for proving the correspondence between QM and NRCPH (see, e.g., [12], pp. 127, 128; [13], pp. 19, 20). Hence, the correspondence between QFT and QM shows that the action S of QFT should be a mathematically real Lorentz scalar. This outcome is consistent with the Weinberg QFT arguments (see, [4], p. 300).

The Lagrangian density (2.1) is still an incomplete description of the state of a given elementary particle. Indeed, the existence of a physical particle is established via a measurement process where the particle affects the time evolution of a measurement device. Form (2.1) of the Lagrangian density comprises the fields and the metric $g_{\mu\nu}$. The metric is used as a dynamical variable for the description of gravitational interaction [1]. Hence, *the Lagrangian density of other interactions requires an interaction term that depends on an external field that carries the interaction.* For example, in the classical theory, the interaction term of a charged particle and EM fields is

$$\mathcal{L}_{int} = -ej^{\mu}A_{\mu}, \tag{2.5}$$

where e is the particle's charge, j^{μ} is its four-current, and the components of A_{μ} are the EM potentials $\phi, \boldsymbol{A}$ (see [1], p. 75; [14], p. 596). Relativistic aspects of A_{μ} are discussed below in chapters 4 and 5.

2.3 The Noether Theorem

The Noether theorem is an important element of theories derived from the variational principle. This theorem shows that if the Lagrangian density (or the Lagrangian) is invariant under a given transformation, then the Euler–Lagrange equations of this theory conserve an appropriate quantity (see, e.g., [4], p. 307; [15], section 13.7; [16] pp. 17–22).

This work examines the Noether expressions for the energy–momentum density derived from a Lagrangian density of the EM fields. Energy is the 0-component of the energy–momentum four-vector (see [1], p. 29), whereas density is the 0-component of a four-vector (see [1], pp. 73–75). Therefore, energy density is the 00-component of a 4-tensor $T^{\mu\nu}$, which is called the energy–momentum tensor. If the Lagrangian density does not explicitly depend on the space-time coordinates $(t, \boldsymbol{x})$, then the Noether theorem provides an expression for this tensor that proves energy and momentum conservation. This energy–momentum tensor is

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial \psi_{,\nu}} g^{\alpha\mu} \psi_{,\alpha} - g^{\mu\nu} \mathcal{L} \tag{2.6}$$

(see [1], p. 83; [17], p. 310). The components $T^{\mu 0}$ are energy and momentum density, and this relation proves their conservation:

$$T^{\mu\nu}_{\ \ ,\nu} = 0. \tag{2.7}$$

This book regards the Noether theorem as an important element of theoretical physics. Thus, it carries out a close examination of the Noether theorem and applies the self-evident requirement stipulating that the Lagrangian density and the transformation must take a mathematically coherent form.

2.4 Wigner's Analysis

Relying on mathematical arguments, the Maxwell equations can be identified as a precursor of SR. Thus, Maxwell recognized that free EM waves are a solution to the homogeneous form of his equations. An important property of these waves is that they travel at the speed of light in every inertial frame. This is a fundamental property of SR. Einstein's formulation of SR was a breakthrough in the physical concept of the structure of the universe. In this theory, physical variables take the form of four-dimensional tensors that are defined in Minkowski space. Lorentz transformations induce boosts and rotations in this space, and such transformations make up a six-parameter group called the Lorentz group. An addition of space-time translations to the Lorentz group creates a 10-parameter group called the Poincaré group (also called the inhomogeneous Lorentz group).

Wigner analyzed the unitary representations of the inhomogeneous Lorentz group [18]. His work demonstrates far-reaching mathematical arguments that describe particle properties. Sternberg used these words as a description of the remarkable significance of Wigner's work:

> It is difficult to overestimate the importance of this paper, which will certainly stand as one of the great intellectual achievements of our century (see [19], p. 149.)

In addition to Wigner's original work, his analysis can be found in textbooks, such as in [19, 20]. The results of his work used later in this book are as follows:

Wig.1 A quantum state has well-defined mass and spin.

Wig.2 There are two categories of physically meaningful quantum systems: The first category comprises systems with mass $m > 0$. The velocity of particles of such a system is smaller than the speed of light. A member of this category has a well-defined spin, and its j_z takes $(2j+1)$ different values. The second category comprises massless particles, where $E^2 - p^2 = 0$ and $E > 0$. These particles travel at the speed of light. Instead of spin, these particles have 2 degrees of freedom of helicity.

Wigner's work is a purely mathematical analysis. It provides an impressive example of the powerful merits of mathematics and the physical relevance of results that are derived from its application. Wigner uses fundamental elements of a quantum theory and SR, which is a theory constructed on two principles–namely the principles of relativity and the constant speed of light. One may wonder how these principles can affect the apparently irrelevant property of the number of spin projections on the z-axis; because of this issue, it is quite surprising that Wigner proved that the J_z of massive particles has $(2J+1)$ m-values. Nevertheless, a spin $= 1$ photon has only two values ± 1 of its polarization. Experiments support these claims.

Conclusions:

W.1 If two physical entities do not have the same mass, these entities are different. In particular, a massive particle and a massless particle are different physical entities.

W.2 If two physical entities do not have the same spin, these entities are different. In particular, the spin projection of a spin $S \geq 1$ of a massive particle has $2S+1 \geq 3$ possible values. In contrast, the spin projection of a spin $S \geq 1$ massless particle has only two possible values. This means that massive particles and massless particles are inherently different physical entities even if they have the same total spin.

W.3 The energy of a physical particle is greater than zero, and its energy–momentum invariant $E^2 - \boldsymbol{p}^2$ is non-negative. This means that the energy–momentum four-vector of a genuine physical particle cannot be spacelike, and its energy should be greater than zero.

Unfortunately, Wigner's work is not adequately discussed in many QFT textbooks (see, e.g., [4], p. 1). In contrast to this situation, Wigner's conclusions are seriously examined here. His work belongs to the quantum domain. Therefore, its meaning is used here as a supporting argument that justifies some of the results.

2.5 Two Electromagnetic Theories

Every physical theory takes the form of a mathematical structure that adequately describes experimental data. The mathematical elements of a physical theory have the general structure of a mathematical theory – several axioms are regarded as correct and derivation laws provide other elements of the theory. Therefore, the set of axioms are the foundation of any given theory. In principle, two different sets of axioms may provide theories that yield the same prediction of measurements. Classical electrodynamics is an example of this kind.

An examination of textbooks on classical electrodynamics indicates that there are two approaches to this topic. One approach uses Maxwell equations as the equations of motion of EM fields, and the Lorentz law of force as the force exerted on a classical charged particle. Thus, it is stated that

> Maxwell equations form "the basis of all classical electromagnetic phenomena. When combined with the Lorentz force equation and Newton's second law of motion, these equations provide a complete description of the classical dynamics of interacting charged particles and electromagnetic fields" (see, [14], p. 218).

The first eleven chapters of [14] describe this theory. Here, this theory is called Maxwell Lorentz electrodynamics (MLE). The books [21, 22] also provide examples of MLE. On the other hand, in [1], Landau and Lifshitz derive the laws of electrodynamics from the variational principle (also called the principle of least action). In particular, their textbook applies the variational principle and derive Maxwell equations and the Lorentz force (see chapters 3,4 of [1]). Here, this theory is called variational electrodynamics (VE). Landau and Lifshitz [1] use a Lagrangian for charged particles and a Lagrangian density for EM fields.

The role of the potential A_μ highlights a crucial difference between MLE and VE. Indeed, the fundamental equations of motion of MLE are Maxwell equations (see [1], chapter 4) and the Lorentz law of force (see [1], eq. (17.5)). These equations are *independent* of the potentials A_μ. In contrast, in VE, the classical action of Maxwellian electrodynamics is (see [1], p. 75)

$$S = -\sum \int m\,ds - \int A_\mu j^\mu \, d^4x - \frac{1}{16\pi}\int F_{\mu\nu}F^{\mu\nu}\, d^4x. \qquad (2.8)$$

Here, s is the invariant interval (see [1], p. 5), while the components of $F^{\mu\nu}$ are

$$\begin{aligned} F^{\mu\nu} &= g^{\mu\alpha}g^{\nu\beta}(A_{\beta,\alpha} - A_{\alpha,\beta}) \\ &= \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}. \end{aligned} \qquad (2.9)$$

(see [1], p. 65). The interaction term of the action of (2.8) explicitly depends on the potentials A_μ. Landau and Lifshitz prove that the Euler–Lagrange equations of the classical version of VE are the Maxwell equations and the Lorentz force [1]. Hence, MLE and VE describe the same kind of interacting objects. However, VE depends on the potentials, whereas MLE is independent of potentials. This state of affairs justifies a separate analysis of the role of the potentials in both MLE and VE. If the potentials play the same role in MLE and VE, then a coherent mathematical analysis of this kind should prove this property. This means that, in this case, a coherent analysis can only illuminate several aspects of this topic.

This book proves that MLE and VE are different theories. In particular, a gauge transformation of the EM potentials does not alter the EM fields. Hence, MLE is invariant under a gauge transformation. The explicit calculations of chapter 5 closely examine the mathematical coherence of gauge transformations. These calculations prove that although the VE Lagrangian density is invariant under a gauge transformation, the entire VE theory is *not* invariant under these transformations. This is an important outcome because it contradicts the relevant claims of most other contemporary textbooks, if not all of them.

2.6 An Elementary Particle

This section discusses the concept of elementary pointlike particle (EPP). For the sake of brevity, the acronym EPP denotes both the singular and plural forms of the term *elementary pointlike particle.* Several arguments support the concept of an EPP.

PL.1 Landau and Lifshitz show two different arguments that prove that the classical sector of SR requires an elementary particle to be pointlike (see [1], pp. 46, 47). Their proofs examine the possibility of an elementary particle whose volume does not vanish. This particle should be absolutely rigid because its elementary nature means that it cannot comprise distinct components that may move with respect to one another. The authors of [1] prove that SR denies the absolute rigidity of such a particle, concluding that

> within the framework of classical theory elementary particles must be treated as points (see [1], p.47).

PL.2 Basic QFT arguments arrive at the same conclusion. Here, the fundamental QFT equations are derived from a Lagrangian density with the form

$$\mathcal{L}(\psi, \psi_{,\mu}) \tag{2.1}$$

(see, e.g., [4], p. 300). Here, $\psi(x)$ and its derivative $\psi_{,\mu}(x)$ depend on x, where $x \equiv (t, \boldsymbol{x})$ denotes the four space-time coordinates. (The present book follows this practice, and in many cases the function argument x denotes the four space-time coordinates.) It is stated there that

> All field theories used in current theories of elementary particles have Lagrangians of this form.

The four independent coordinates of $\psi(x)$ mean that this function can describe properties of the quantum particle at the space-time point $(t, \boldsymbol{x})$. However, to describe of a composite particle at space-time points near $(t, \boldsymbol{x})$ it is necessary to include a greater number of independent variables that also account for the specific spatial structure of the given composite particle. Hence, QFT theories of a function $\psi(x)$ apply to an EPP. The argument (see R.2, p. 6) indicates the validity of this claim in the classical domain. This outcome is consistent with the classical proof of the previous item.

PL.3 Experiments provide impressive support for the pointlike attribute of an elementary particle. For example, protons and neutrons are composite particles that comprise quarks. The radius of these particles is about 10^{-13} cm [23]. In contrast,

> the electron is an elementary particle, and it has no specific size. The present experimental upper bound of the electron's radius is about seven orders of magnitude smaller than the proton's radius [24]. This information can be regarded as a remarkable support for these elements of theoretical physics.

This section explains the *pointlike* attributes of an elementary particle. This issue holds both in classical physics and in QFT. Unfortunately, it is still not widely recognized. For example, the pion is a quark–antiquark bound state, and its spatial size is nearly the same as that of the proton [23]. Hence, contrary to the claims of some publications, a pion *cannot* be the Yukawa particle $\phi(x)$ that mediates nuclear interactions.

> It can be stated that the literature does not contain a serious objection to the principles that are pointed out above. Every specific analysis that is described in the following chapters relies of these principles and on the fundamental requirement of the fit of every physical theory to experimental results that belong to the appropriate domain of validity.

Chapter 3

Electromagnetic Fields (CS)

Classical electrodynamics is a theory that describes the time evolution of two physical entities – EM fields and electrically charged particles. This chapter examines several properties of EM fields. Maxwell equations are the equations of motion of the EM fields (see, e.g., [1], chapter 4; [14], pp. 217–219). Their relativistic covariant form is (see [1], pp. 71, 79; [14], p. 551)

$$F^{\mu\nu}_{\ ,\nu} = -4\pi j^{\mu}, \quad F^{*\mu\nu}_{\ ,\nu} = 0, \tag{3.1}$$

where $F^{\mu\nu}$ is shown in (2.9), $F^{*\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$, and j^{μ} is the four-current of the electric charge. This chapter discusses physical aspects of these fields.

3.1 Radiation Fields and Bound Fields

The standard form of Maxwell equations (3.1) has one inhomogeneous tensorial equation and one homogeneous tensorial equation. The special case where all Maxwell equations are homogeneous can be expressed as follows:

$$F^{\mu\nu}_{\ ,\nu} = 0, \quad F^{*\mu\nu}_{\ ,\nu} = 0. \tag{3.2}$$

Solutions of the homogeneous Maxwell equations (3.2) are called *radiation fields* (see, [1], p. 116), while solutions of the inhomogeneous Maxwell equations (3.1)—where the four-current $j^{\mu} \neq 0$ at some space-time coordinates—are called *bound fields.* Consider the following assertion:

Assertion: Radiation fields and bound fields represent entirely different physical entities.

The full consequences of this assertion are presently (June 2022) ignored by the general community. Furthermore, this assertion has far-reaching consequences, as explained below. Therefore, several independent proofs substantiate it.

RB.1 A significant property of electrodynamics is the quadratic dependence of the energy–momentum tensor on the fields (see [1], pp. 86–89). Let us compare the role of this tensor in the case of radiation fields with that of bound fields.

Consider the case of incoming radiation that interacts with the charges of an antenna. This effect is used in radio receivers, cell phones, GPS locations, and so on. Here, the incoming radiation carries energy and momentum that is organized in a form with specific information. The possibility of detecting radiation by several receivers is an important property of this object. Radio receivers illustrate this point. (In cell phones, a specific device disables this possibility.) This effect proves that radiation is an independent physical object that may be *detected* by the charges of a measurement device.

Let us examine the state of two charged particles that are close to each other. Bound fields of each particle exert force on the other particle. In this case, the energy and momentum of the fields of the *interacting charges* represents the energy and the momentum that each of these charges emits/receives. The validity of this assertion relies on the following properties of electrodynamics:

PED.1 As stated above, the electromagnetic energy–momentum tensor depends quadratically on the fields. In the case of radiation, these fields are independent of local charges. For bound fields, this tensor depends on the fields of the two interacting charges.

PED.2 The force exerted on a given charged particle is linear in the external field *and* in the charge of this particle (see [1], p. 51).

PED.3 The energy–momentum tensor of the fields exchanges energy and momentum with charges. The combined system of the fields and the two massive charged particles conserve energy and momentum (see [1], pp. 88, 89).