FUNDAMENTAL CONCEPTS OF PHYSICS

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To Barbara, the most important person in my life.

PREFACE

Does the world really need another conceptual physics book? This is a question I asked after teaching the standard one semester course to non-science undergraduates for the past four decades. After examining the available texts, I concluded there was a space for a relatively small, inexpensive text that would present the science of physics in a fashion that did not emphasize the mathematical or computational aspects of the subjects, but instead attempted to introduce some of the reasoning used in the past to arrive at our present understanding of the workings of the physical universe. The following text represents my effort to fulfill this need.

The text does not present the material of physics in the same way we present it to science and engineering students for two important reasons. We train science and engineering students to view physical situations with an analytical eye so that they obtain a quantitative understanding of the situation and are able to make quantitative predictions of observable results arising from the situation. These students think in an analytical mode. Non-science students need not approach physical reality as a series of situations to be analyzed, but more as a mosaic of natural parts forming a whole picture of our world and its place in the universe. In some respects, we expect our technical students to follow the thought processes of an Aristotle while our non-technical students follow those of Plato. Both paths are valid.

You will find very few equations, numerical examples, or problems in this text. Those that are included are for the benefit of the student who appreciates the economy of expressing truths in symbolic mathematical representations and for the instructor who might want to explore some quantitative expectations of the science. The student who is not mathematically inclined need not fear being overwhelmed with numbers.

The included illustrations are simple line drawings by design. While multi-colored artistic representations are useful in many instances, the essence of a concept can often be shown with a minimum of detail. Including only the essential features of a pictorial representation of some physical phenomenon often allows students to see beyond extraneous complications and obtain understanding of the underlying physical situation.

The book is divided into fifteen chapters. Each chapter attempts to introduce large concepts that are applicable to several areas of the physical world. For instance, the concept of momentum and the conservation of momentum in the absence of external forces is central to our understanding of phenomena ranging from the collision of two bodies to the pressure exerted by a confined gas made up of countless particles. For our technical students we would examine these two situations at different times. However, the concept of momentum conservation is central to both situations. For students using this text, understanding of the central concept is more important than the ability to apply the concept to a specific situation where quantitative information is required.

I hope students will approach reading this book more as an intellectual adventure than as an academic chore. When the course is over, if students have a greater understanding of the workings of the universe in which we are all a part and the intellectual developments that lead to this understanding, the book will have been successful.

Good luck in your study of the concepts of physics. I hope you develop a familiarity with the concepts and those who were responsible for their discovery.

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CHAPTER I

PLANETARY MOTION AND UNIVERSAL GRAVITATION

Since the beginning of recorded history, humanity has expressed an interest in the physical world. Individuals attempted to explain phenomena observed in this world on the basis of discovering the causes of the noted effects. The basic assumption was that every effect had a findable cause. This assumption of causality is the basis of our knowledge of the physical universe.

In the earliest recordings of the history of the human endeavor we now call science, individuals had recourse only to superstition and the invocation of supernatural powers. As knowledge and sophistication improved over the course of centuries, supernatural explanations were rejected, as natural laws became known. The object of the study of physical science is to describe observed phenomena in terms of basic, fundamental, universally operative natural laws.

In attempting to attain this objective, the physical scientist reasons in either an inductive (synthetic) or a deductive (analytic) fashion. The former method attempts to first propose universal laws that can be applied to specific problems or phenomena. This is the method often used today in theoretical investigations. The theorist attempts to discover fundamental laws and then tests the validity of these laws by comparing their predictions with the results of experiments or observations. The latter method reverses the steps in that the scientist first experimentally observes a phenomenon or a set of phenomena, then attempts to develop a theory that explains these observations. Both methods are valid and useful in obtaining a general view of nature.

Because both the deductive and the inductive reasoning schemes are valid, it is useful to investigate examples of their application in arriving at our currently accepted ideas on the nature of physical reality. A useful example is the development of the explanation of the problem of planetary motion.

It is likely that the problem of explaining the motion of planets in the night sky is one of the oldest to which humanity gave any attention. Ancient civilizations gave much thought to the problem of the objects wandering through the heavens and arrived at a wide variety of explanations for the observed phenomenon. Some of their explanations seem almost comical today but are, in fact, attempts to explain one of the most fundamental and important mysteries of the ancient world.

The problem of wandering stars received much attention by all the early civilizations, but the explanation stated by the philosopher Ptolemy is perhaps most important because it influenced thinking in Europe for centuries. Ptolemy described the observed universe in terms of a model where Earth stood at the center of a number of spheres of increasing diameter. The moving heavenly bodies, sun, moon, planets, were constrained to move within the spheres on circular paths about Earth. The stars were said to rest on the sphere with the largest diameter and were thought to be the dome of heaven.

Ptolemy's ideas were not seriously challenged until the beginning of the Renaissance. One reason for their long acceptance is that his theory is logically consistent and was not subject to experimental or observational verification until adequately refined techniques and equipment were developed in the fifteenth century. Until that time, logical consistency was sufficient for the acceptance of a theory. The idea that observation and experimentation needed to verify a theory is a relatively recent concept.

Once it became possible to make precise determinations of planetary positions, interest in the problem of planetary motion revived. Tycho Brahe, the Danish astronomer, brought the art of astronomical observation to a zenith in the sixteenth century. Brahe was a firm believer in the Ptolemic system that Copernicus was attacking. He felt accurate observations of planetary motion would verify Ptolemy's idea. With the backing of King Frederick II of Denmark, Tycho built the world's finest observatory where he amassed a huge amount of extremely good data on planetary position as a function of time. Unfortunately, Tycho was a better observer than an interpreter and died before he was able to draw any conclusions from this effort.

Johannes Kepler, an assistant of Tycho, obtained possession of Tycho's data trove after his death. With the advantage of having the best observational data ever obtained, Kepler attempted to make the data fit with Ptolemy's concepts. After much effort, he was forced to conclude the theory of Ptolemy had to be abandoned. Only Copernicus' concept of a sun-centered solar system fit the observational data.

In Copernicus' scheme, Earth assumes a position of being only one of several planets traveling around the sun. This displacement of the centrality of Earth in the heavens did not receive ready acceptance from the religious and political authorities in Europe at the time and it was some time before the Copernicus explanation found acceptance. Now, however, we are so sure of its validity that it serves as the basis of our ability to send space probes to other bodies contained within our solar system.

Kepler was an empiricist in that he did not produce any data, but used the data produced by someone else to deduce mathematical relations among the various parameters of interest in the problem. While he did not have a fundamental theory to explain his discoveries, he was highly successful in his endeavor. He was able to compress the data Brahe had complied over a lifetime of observations into three compact laws describing the motion of the planets. Kepler's first law describes the paths of the planets. Simply stated, it is:

The planets move in elliptic orbits with the sun at one focus of the ellipse.

This law is illustrated in the following Figure 1.

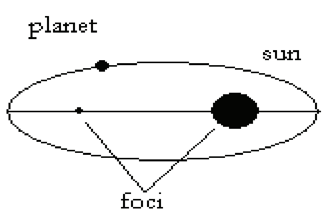

Figure 1.

Because the planet moves in an elliptical path, the distance from the sun to the planet is not constant, but varies with time. This conclusion differed from the theory of Copernicus or Galileo who assumed the planets moved in circular paths about the sun. The observational data required a modification to one of the points of the heliocentric theories, but showed those theories to be correct in their essential conclusion that the system was a sun centered rather than an earth centered entity.

Based on his empirical interpretation of the data, Kepler deduced his second law of planetary motion. This law states:

The line drawn from the sun to a planet would sweep out equal areas in equal times.

To illustrate this law consider Figure 2.

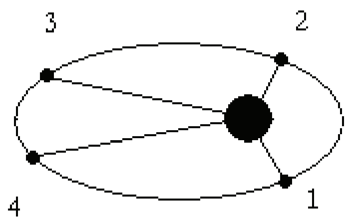

Figure 2.

If the planet is in position 1 at some point in time, after a certain time passes, say for sake of argument two weeks, it is in position 2. After several weeks have passed, the planet is now in position 3. Again after two weeks, the planet has moved to position 4. The line joining the planet to the sun has swept out equal areas when the planet moved from position 1 to 2 as it did when the planet moved from position 3 to 4. Casual observation leads us to the conclusion that the planet was traveling faster when it was nearer the sun than it was when it was farther away. Kepler noted this fact, but needed ten more years of study before he was able to come to a quantified statement concerning it.

Because it is now apparent the motion of planets in their orbits is not one where they travel equal distances in equal times, we need to develop some quantitative understanding of exactly what is meant when we discuss an object's speed or velocity. Although we commonly use the terms speed and velocity interchangeably, the scientist is very specific in differentiating these two concepts. To a scientist, such as Kepler or Galileo, an object's speed is a measure of how fast it moves regardless of direction, while velocity is concerned both with how fast it moves as well as its direction. Quantities such as speed that can be completely specified by a magnitude are named *scalar* quantities, while those needing to be specified by both a magnitude and a direction are called *vector* quantities. There are many examples of both scalar and vector quantities that you will encounter in your study of physical science.

Consider two points, A and B, separated by two meters in an east-west direction. The scalar distance between them is two meters, while the vector displacement from point A to point B is two meters in the west direction. If an object travels from A to B in four seconds, its scalar speed is ½ meter per second, while its vector velocity is ½ meter per second to the west.

As you can see, distance is the scalar magnitude of the displacement vector and speed is the scalar magnitude of the velocity vector. The average velocity to go from A to B is ½ m/s, but that does not mean that the object's speed was ½ m/s at every point on its travel from A to B. It is possible that it took three seconds to go half the distance and another one second to travel the other half. To have a real understanding of the object's motion we need to have a measure not only of its *average* velocity of ½ m/s west, but of its *instantaneous* velocity at every point of its journey. This need led Newton and Leibnitz to develop the mathematical discipline of differential calculus.

Because a vector quantity changes if either its magnitude or direction changes, when we talk about the rate of change of the velocity vector with respect to time that we call *acceleration*, we can focus on the acceleration associated with a change in the direction of the velocity vector, or the acceleration associated with a change in its magnitude. We define *tangential* acceleration as the rate of change of the magnitude of velocity and the *radial* or *centripetal* acceleration as the rate of change of the direction of velocity.

Returning to Kepler's analysis of Brahe's data, we can see the velocity of the planet in its motion about the sun continuously changes in both direction and magnitude. Kepler worked for ten years after he published his first two laws of planetary motion to arrive at some mathematical relationship that would account for the continuous variation of the planets' orbital velocity. He was finally able to relate the time it takes a planet to complete its orbit about the sun, its *period*, to the average distance between the planet and the sun. This he published as his third law of planetary motion: *The ratio of the cube of the distance between planet and sun to the square of the period of its orbit is constant.*

We often find it convenient to write laws such as Kepler's third law in mathematical formulations. If we define the average distance between sun and planet by the letter, r, and the period of its orbit by, T, we can then write this law as:

$$r^3 \propto T^2$$

Example 1. The planet Jupiter takes 12 years to complete one orbit. How far is Jupiter from the sun?

Using Kepler's third law, we can write the ratio of the cube of Jupiter's average distance from the sun to the cube of Earth's average distance equals the ratio of the square of Jupiter's period of revolution to the square of Earth's period.

$$r^3_J/r^3_E = T^2_J/T^2_E$$

Knowing the average distance between Earth and the sun is 93,000,000 miles, we can state the cube of Jupiter's distance from the sun is

$$r^3_J = 12^2/1^2 \text{ (93,000,000 miles)}^3.$$

Or, Jupiter's average distance from the sun is about 487,000,000 miles.

Kepler's three empirical laws of planetary motion represent his unique contribution to our knowledge. It is a tribute to Kepler's energy and genius that the laws of planetary motion carry his name. Remember, Kepler discovered his laws by empirical study. He had masses of very good observational data available to him from which he could glean mathematical relationships. He did not, however, have any fundamental ideas of the forces in nature involved in causing the planets to move in the paths he so completely described.

The study of planetary motion was not completed by Kepeler's publication of his three laws.

The next great advance in developing our understanding of planetary motion came from Sir Isaac Newton. Newton studied the motion of objects in order to discover the causes of motion. His approach, the description of an object's motion based on the knowledge of the causes of changes in motion, is called *dynamics*. If the motion is described without regard to the causes of change, the study is called *kinematics*. Kepler described the kinematics of planetary motion. Newton described the dynamics of planetary motion.

Newton was neither an observer as was Brahe, or an empiricist as was Kepler. Instead, he was a theorist. His goal was to put the study of the motion of the planets on a firm theoretical basis. In order to accomplish this, he needed to develop certain mathematical tools and techniques and discover laws that were universally applicable to the motion of material bodies.

Until Newton's time, mathematic relationships were static and expressed truths between variables that held for all time. Newton, realizing that variables could change with time, developed the differential calculus to describe the rate of change of one variable with respect to another.

To illustrate Newton's thinking, consider again our discussion of the velocity of a particle as it moves from point A to point B as shown in Figure 3.

Figure 3.

The magnitude of the average velocity for the particle to move from point A to point B is the ratio of the distance between the two points to the time it takes to go from one point to the other. Now consider the velocity to move from point A' to point B'. The distance between the points has changed and the time needed for a particle to move from one to the other has changed. If, however, the ratio of distance to time has not changed, the average velocity of the particle remains constant. In general, that ratio is different in the first case than in the second. In fact, average velocity can continuously differ along a path leading from one point to another. In order to overcome this difficulty, Newton argued that at every point of an object's journey, a particle had a unique, determinable *instantaneous* velocity.

As we have already seen, the velocity of a planet in its orbit about the sun is not constant but changes with time. Therefore, any completely rigorous mathematical treatment of the problem of planetary motion must, of necessity, use the calculus as developed by Newton.

However, Newton realized that merely having the mathematical tools to describe planetary motion was not enough. He needed to possess an understanding of the fundamental laws of motion of any body. Fortunately, some of these fundamental laws were already known.

Perhaps the most fundamental law concerning the motion of a body is the law of inertia. *A body's inertia is its resistance to a change in its motion.* Newton knew the concept of inertial that had been developed over time and was succinctly stated by Galileo. Before Galileo, it was generally thought that a continuous force was needed to maintain a body's motion. Galileo pointed out that in the absence of any external force acting on a body, its motion would remain constant. Newton appreciated Galileo's thought and stated it in his *First Law of Motion: A object at rest will remain at rest and an object in motion will remain in motion in a straight line with constant speed unless acted upon by an external force.*

Based on this statement of his First Law, Newton realized the planets were under the influence of some external force. To describe planetary motion, Newton realized he needed to answer two questions. How does force acting on a body affect the motion of a body? What is the nature of the force between the sun and a planet?

He answered the first question with his *Second Law of Motion: When acted upon by an external force a body is accelerated in the direction of the force.* Let's consider the implications of this statement. If I take a block of wood and let it drop, is a force acting on it? By Newton's

Second Law, we must say there is a force acting vertically downward that changes the block's velocity from zero when I first release it to some increased value when it hits the floor. If I slide that same block of wood along a tabletop does its motion change? Certainly it starts with an initial velocity but stops after a short time. A force must have acted to slow it to a stop.

Newton needed to arrive at a quantitative understanding of the exact nature of the acceleration imposed on an object by the application of an external force. We can attempt to quantify our ideas by considering the pushing of the block across the tabletop. If we lay down a layer of oil before we push the block, we find the block tends to remain in motion much longer than when the tabletop was dry. If we go farther and drill small holes in the tabletop through which we blow compressed air on which the block floats we find the block once set in motion tends to continue in motion with undiminished speed for a very long time. Let's apply varying forces to the block described in the last case and observe its acceleration. We note when we double the force we double the acceleration as shown in Figure 4.

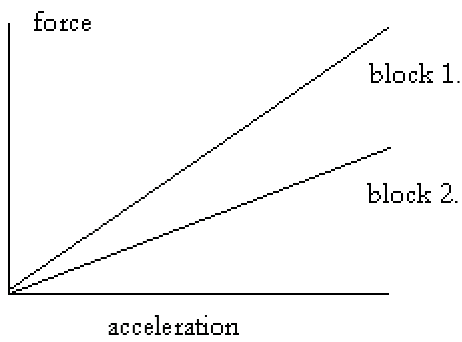

Figure 4.

If we take a second block of the same wood that has twice the volume as the first block, we note the acceleration produced by a given amount of force is just half its value in the first case. We conclude, therefore, there is some intrinsic property of material that represents the relationship between the acceleration an object acquires and the force that produces it. This property of material is called its *mass*. Mass is a measure of the amount of material a body possesses or is a measure of the body's inertia. The greater an object's mass, the greater force must be applied to cause a given acceleration. In a mathematical statement we can write,

$$\mathbf{F} = m\mathbf{a} \qquad \text{Equation 1.}$$

This equation is the quantitative statement of Newton's Second Law of Motion. To be useful, however, we need to define a standard of mass to which any object can be compared. Such a standard is the mass of a platinum-iridium cylinder maintained by the International Bureau of Weights and Measures in France. The mass of this cylinder is defined to be one *kilogram*. With this definition, we can state that a mass of one kilogram will experience an acceleration of one meter per second squared when acted upon by a certain force. This amount of force is called a newton in honor of Sir Isaac.

Example 2. What is the ratio of the acceleration given to an object containing two kilograms of mass to that given to a ten kilogram object when both are subjected to the same force?

Because the force acting on the two objects is the same, the ratio of their accelerations will be inversely proportional to the ratio of their masses

$$a_1/a_2 = m_2/m_1.$$

Therefore, the ratio of the acceleration of the less massive object to that of the more massive object will be 10/2. The less massive object will accelerate five times faster than the more massive one.

In the same manner that we defined a standard unit of mass, we define standard units of length and time. Until 1960, the standard unit of length, the *meter*, was defined as the distance between two scratches on a platinum-iridium bar. Since that time, we defined a much more accurate atomic standard. Likewise, the standard unit of time, the *second*, was defined until 1967 as a certain fraction of a mean solar day. We have also replaced this definition with a more accurate atomic standard.

Concerning the force on the planets causing them to describe paths about the sun, Newton concluded that the force of the sun on the planet must be equal and opposite to the force of the planet on the sun. Generalizing this line of thinking, Newton stated his *Third Law of Motion: For every active force there is an equal and opposite reactive force.*

The force of attraction between the sun and the planets is the same type of force that exists between the earth and the moon, or, in fact, between every pair of massive objects in the universe. This universal force is the force of *gravity*.

Newton, having defined the basic laws concerning the action of forces on objects that we now call *mechanics*, was in a position to investigate the particular nature of the gravitational force required to ensure planets moved in the elliptical orbits described by Kepler. He concluded there exists the *Law of Universal Gravitation*. Simply stated, this law maintains there is an attractive force between every pair of massive objects in the universe that varies as the product of the masses involved and inversely as the square of the distance between them. Written in mathematical shorthand, this is:

$$F \propto m_1 m_2 / r^2.$$

If we introduce a constant of proportionality, we can write this law in an equation:

$$F = G m_1 m_2 / r^2. \qquad \text{Equation 2.}$$

We use the letter, G, as the symbol for the universal gravitational constant. In the system of units we have been discussing, the SI system, the value of G is exceedingly small.

Newton published his law in 1684, but it was more than a century later that Henry Cavendish performed the experiment that determined the numerical value of G. Cavendish published the results of his experiment in 1797. The currently accepted value of the universal gravitational constant is:

$$G = 6.67 \times 10^{-11} Nm^2/kg^2.$$

This number is one of the most fundamental constants in the universe.

You may have heard the romantic story that Newton saw an apple fall and was led to developing his theory of universal gravitation. This story is probably much too romantic to be true. In reality, Newton developed his theory by observing the motion of the moon about the earth and by applying general laws of motion used in conjunction with the mathematical tool of the calculus that he invented. With this analysis, Newton was able to derive Kepler's laws of planetary motion from universally applicable fundamental laws of nature. This is the essence of science, the discovery of natural laws that can account for the ways things work.

Newton's universal gravitation theory stood the test of time for more than two hundred years. It was not until the first part of the twentieth century that his law had to be modified to account for observed results that differed from theoretical prediction. The amount of difference between predictions made using Newton's formulation and the observed results were exceedingly small but were larger than observational error.

Albert Einstein in his *General Theory of Relativity* published in 1915, produced the next great advance in our understanding of gravity. Einstein's theory links the concept of gravity to geometry and will be discussed later in this book. For now, we can say that the predictions of Einstein's theory agree with observations, and we believe the problem of planetary motion is completely solved.

We should now ask ourselves what we have learned from our discussion of the problem of planetary motion. On a shallow level, we learned how one problem developed from observation, to empirical rules, and finally to a theoretical understanding based on fundamental universal physical laws. As we go on we will see examples of other problems that have been solved in the reverse manner, i.e. a theoretical law used to make predictions that are verified by observation or experiment.

On a deeper level, we discovered the force of gravity with which we are intimately associated in our daily lives. We have seen how this force influences the motion of heavenly bodies. In the next chapter, we will investigate gravity closer to ourselves.

QUESTIONS

1. What was Tycho Brahe's contribution to the study of planetary motion? Kepler's? Newton's?
2. What are Kepler's three laws of planetary motion?
3. How does a vector quantity differ from a scalar quantity?
4. What do we mean by velocity? acceleration?
5. How does a centripetal acceleration differ from a tangential acceleration?
6. What is the difference between an instantaneous and an average velocity?
7. What do we mean by "inertia"?
8. What is a body's mass?
9. How does the calculus differ from ordinary algebra?
10. What are Newton' three laws of motion?
11. What was Henry Cavendish's contribution to our understanding of the force of gravity?
12. State Newton's Law of Universal Gravitation.
13. How would things differ if G, the universal gravitation constant, were not such a small number?
14. What contribution did Einstein make to the concept of gravity?
15. Why is it necessary to define standards of length, mass, and time?
16. The United States uses the British system of measurements. What is the unit of length in the British system?
17. List some quantities that are scalar i.e. are completely specified by a magnitude.
18. List some quantities that are vector in nature.

CHAPTER II

QUANTIFICATION AND MECHANICS

In the previous chapter, we were able to solve a specific problem concerning the motion of massive bodies by the application of certain general principles. If these principles really are general, we should be able to apply them to all problems where we attempt to describe the motion of massive objects. This is the goal of *mechanics*.

As with the rest of physics, mechanics seeks to describe phenomena in quantitative terms. In order to achieve this goal, we need to answer two fundamental questions:

1. What are the essential quantities to be measured?
2. What units will be used to measure these quantities?

In the study of mechanics, the essential quantities are length, mass and time and the units we use are defined by comparison with the standards presented in the previous chapter. The standard of length is the meter, for mass it is the kilogram, and for time the second. All other quantities that concern us are derived from these fundamentals. Velocity is simply a measure of the ratio of distance to time, acceleration is the ratio of distance to time squared. Force is the product of mass and the ratio of distance to time squared.

The units we use to measure fundamental quantities are somewhat arbitrary. We generally agree to use the SI system of units discussed previously. However, some older texts use a system of units where the basic unit of length is the centimeter, mass is measured in grams, and time in seconds. This system of units is often referred to as the *cgs* system. Because the cgs system was in general use before we standardized on the SI system, many quantities are still commonly measured in these units. For instance, fluid volume is often measured in cc's or cubic centimeters and magnetic fields are often specified in gauss. This is particularly true in the medical profession.

Another system of units commonly used in the United States is the British system. In this system, weight, which is the force of gravity acting on a mass, is used. In this system, length is measured in inches, feet, yards, rods, chains, miles, and furlongs. Weight is defined in ounces, pounds, and tons. While time is measured in seconds, minutes, hours, days, fortnights, months, and years. In the British system, a good unit for velocity would be furlongs per fortnight, although this is rarely used. We commonly measure amounts of floor covering in square yards, the amount of gasoline we purchase in gallons, and the amount of heat generated by a propane grill in BTU (British Thermal Units) per hour.

In general, we will use the SI system in our discussion of mechanics. However, from time to time, when it is convenient, we may use either cgs or British units.

Consider the relationship between an applied force and the resultant acceleration it produces on a massive object. In Figure 1, the force will be measured in newtons and the acceleration in m/s^2.

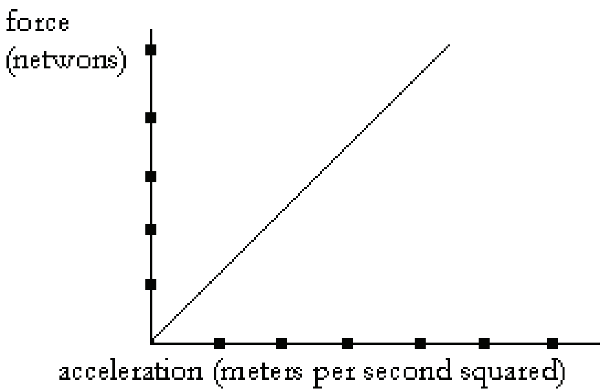

Figure 1.

The newton is the unit of force equal to the product of one kilogram and one meter per second squared. Thus, we have quantified our concept of force to indicate that one newton of force acting on a one kilogram object will impart an acceleration of one meter per second squared. We are now able to ask questions such as the following. What force is needed to accelerate a mass of four kilograms by three meters per second? Or, what acceleration will a six kilogram mass receive when acted upon by a ten kilogram force?

In Figure 1, each tick on the horizontal axis represents an additional acceleration of 2 m/s^2, and each vertical tick represents a force of an additional 5N. Therefore, we can read off the graph a force or 10N produces an acceleration of $4m/s^2$. Thus, we can conclude the mass being accelerated is 2.5 kg.

You might question the reason for bothering with this level of quantification when we stated we wished to discuss the concepts of physics, not its mathematical computations. The answer is twofold. In the first place, physics itself is a quantitative study. The concepts that are involved are those concepts that lead to quantitative, numerically expressible predictions on the behavior of the matter under study. The heart of the study of physics is often expressed by a quotation of Lord Kelvin that maintains quantitative knowledge is the only legitimate knowledge. I have chosen not to include Kelvin's original statement only because its Victorian language makes it sound pompous and smug. The second reason for introducing this level of quantification is to facilitate the understanding of those ideas that can best be expressed in a quantitative fashion. Often new ideas can be gleaned from old information by investigating the relationships that exist among the quantified variables. Of course, this is the direction taken by Kepler, whose success we noted in the previous chapter.

You may have noticed we used two methods to express the relationship between variables. We either wrote an algebraic equation relating the variables or we drew a graph showing that relationship. When both methods are used to explain a phenomenon, they must contain the same information. This fact was apparent to the French mathematician and philosopher Rene' Descartes, who combined geometric (pictorial) representations with algebraic (numerical) equations to create the study we now call analytic geometry.

Consider again Figure 1 and compare it to the equation representing Newton's Second Law of Motion:

$$\mathbf{F} = m\mathbf{a} \qquad \text{Equation 1.}$$

Figure 1 and Equation 1 contain the same information. The relationship between the applied force and the acceleration it produces is expressed in the equation by the constant, m, while in the graph by the slope of the line i.e. the ratio of the vertical rise of the straight line to its horizontal run.

We can generalize our reasoning to any set of phenomena involving two quantities where one quantity is equal to the product of the other quantity and some constant factor. We call such relationships linear because a plot of one variable vs. the other will produce a straight line. This is true for studies other than those in the physical sciences. Whenever there is a linear relationship between pairs of variables, a change in one will produce a directly proportional change in the other.

We are fortunate if we find a linear relation between variables, but this is often not the situation. Between two variables, say x and y, we may find a relationship that is best represented by a curve as shown in Figure 2.

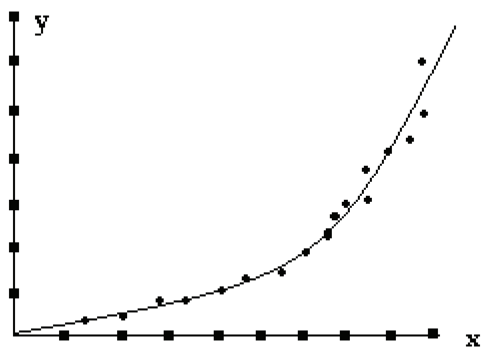

Figure 2

In this situation, the line on the graph best shows the relationship between the variable x and the variable y. We can determine an equation that represents this line, but it will not be a simple statement that y equals the product of a constant factor and x. At best, we may find an equation something like:

$$y = ax + bx^2 \qquad \text{Equation 2.}$$

In this equation, the coefficients of x and x^2 represent constants.

Obviously, we are not able to discuss the slope of a line that is not linear. However, we can look at two points on such a line and connect them with a straight line. The slope of this line is easily