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OPTIMAL ORDERING POLICY FOR DETERIORATING ITEMS IN RESPONSE TO A TEMPORARY PRICE DISCOUNT LINKED TO ORDER QUANTITY AND STOCK-DEPENDENT DEMAND

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Abstract: In this study, the effect of a temporary price discount for a larger order than the regular order offered by the supplier on the retailer's ordering policy is studied. The demand is assumed to be stock-dependent. The units in inventory deteriorate at a constant rate. This study will help the retailer to take the decision whether to adopt a regular or special order policy. The optimum special quantity to be procured is determined by maximizing the total cost saving between the special and regular orders for the cycle time. The algorithm based on analytic results is outlined to take the favorable decision. The numerical examples are given to support the derived results. The sensitivity analysis is carried out to determine the critical inventory parameters.

INTRODUCTION

To boost the demand, increase market share and cash-flow, the supplier offers a temporary price discount to the retailer. For the retailer, the question is to find out whether or not it is advantageous to buy special order at a discounted price or not?" The impact of price discount on the order policy is well cited in review article by Dixit and Shah (2005). The all-unit quantity discounts ordering policy is discussed by Arcelus et al. (2003), Abad (2007), Shah et al. (2005), Bhaba and Mahmood (2006), Abad (2007), Dye et al. (2007), Shah et al. (2008), Mishra and Shah (2009) and their cited references. These formulations assumed that the price discount rate is independent of the special order quantity. However, in market, it is observed that the supplier offers a quantity discount to entice larger orders. For the larger order, the higher price discount rate is offered by the supplier and thereby reducing his inventory. Here,, the retailer has to settle the trade-off for purchase price savings against higher total holding cost.

In addition to holding cost, the retailer incur deterioration cost for products like; dairy products, volatile liquids, blood components, fruits and vegetables, medical accessories etc. The literature surveys by Raafat (1991), Shah and Shah (2000) and Goyal and Giri (2001) advices the retailer to take care of ordering policy while dealing with deteriorating items. One can review Moon et al. (2005), Deng et al. (2007), Liao (2007).

Ouyang et al. (2009) developed a decision policy when a supplier offers a temporary price discount to a retailer for larger order. The price discount rate depends on the units ordered. They assumed that the demand is constant and deterministic.

In this paper, an impact of a temporary price discount on the retailer's ordering policy is studied when the demand is stock-dependent and price discount rate is linked to special order quantity. The units in inventory are subject to a constant deterioration. This analysis will help the retailer to adopt or decline the sales promotional scheme. The retailer's optimal special order quantity is obtained by maximizing the total cost savings between the special and regular orders during a special order cycle time. Two scenarios are discussed : (1) the special order time occurs at the retailer's replenishment time, and (2) the special order time occurs during the retailer's regular cycle time. A computational procedure is outlined to decide to the optimal solutions. The numerical examples are given to support the validated the theoretical results. The sensitivity analysis of the optimal solutions is carried out with respect to the model parameters. The managerial issues are derived.

NOTATIONS AND ASSUMPTIONS

The following notations and assumptions are used to formulate the proposed problem :

Notations

$R(I(t))$	$(= \alpha + \beta I(t))$, stock-dependent demand rate where $\alpha > 0$ is scale demand and $0 \le \beta < 1$ is stock-dependent parameter
C	Unit purchasing cost
A	Ordering cost per regular or special order
I	Holding cost rate per annum
α	Constant deterioration rate; $0 \le \theta < 1$
Q	Order quantity under regular policy
Q^*	Optimal order quantity when regular order policy is adopted
T	Cycle time when regular order policy is adopted
T^*	Optimal cycle time for using a regular order policy
Q_s	Special order quantity at discounted price (a decision variable)
T_s	Cycle time for the special order quantity Q_s
Q	Inventory level before the arrival of the special order quantity; $q \ge 0$
t_q	Cycle time when q – units deplete to zero
T_W	Cycle time for depletion of the inventory level $W = Q_s + q$
$I(t)$	Inventory level at time t when the regular order policy is adopted; $0 \le t \le T$
$I_s(t)$	Inventory level at time t when the special order policy is adopted; $0 \le t \le T_s$
$I_W(t)$	Inventory level at time t when the special order policy is adopted; $0 \le t \le T_W$ where $W = Q_s + q$

Assumptions

(a) The single item inventory system is considered.

(b) The supplier offers the retailer a temporary price discount if the order quantity is larger than the regular order quantity Q^*. The discount rate depends on the quantity ordered and the discount schedule is as follows :

Class	Special order quantity	Discount rate
1	$Q_1 \le Q_s < Q_2$	d_1
2	$Q_2 \le Q_s < Q_3$	d_2
⋮	⋮	⋮
n	$Q_n \le Q_s < Q_{n+1}$	d_n

where Q_k is the k-th discount rate breaking point, $k = 1, 2, \ldots, n+1$ and $Q^* < Q_1 < Q_2 < \ldots < Q_{n+1} = \infty$; d_k is the price discount rate offered by the supplier when the retailer's order quantity Q_s belongs to the interval $[Q_k, Q_{k+1})$ and $0 < d_1 < d_2 < \ldots < d_n$.

(c) The price discount is not passed on to the customers.

(d) Only one time price discount is offered.

(e) The replenishment rate is infinite.

(f) The lead-time is zero or negligible and shortages are not allowed.

(g) The units in inventory deteriorate at a constant rate. The deteriorated units can neither be repaired nor replaced during the period under review.

MATHEMATICAL MODEL

The goal of the study is to decide the advantage of temporary price discount for larger order than the regular order, under the assumption of the stock-dependent demand and constant rate of deterioration of units in retailer's inventory system. If the retailer follows a regular order policy without a temporary price discount, then the inventory depletes in the retailer's inventory system due to the stock-dependent demand and deterioration of units. The rate of change of inventory level can be described by the differential equation

$$\frac{dI(t)}{dt} = -(\alpha + \beta I(t) + \theta I(t)), \quad 0 \le t \le T \tag{1}$$

With boundary condition *I(T)* = 0, the solution of (1) is

$$I(t) = \frac{\alpha}{\lambda}\left[\exp(\lambda(T-t)) - 1\right], \; 0 \le t \le T \tag{2}$$

where $\lambda = \theta + \beta$.

Hence, the order quantity is

$$Q = I(0) = \frac{\alpha}{\lambda}\left[\exp(\lambda T) - 1\right] \tag{3}$$

In this case, the retailer's total cost per order cycle is $A + CQ + Ci\int_0^T I(t)dt$. i.e.

$$A + \frac{C\alpha}{\lambda}\left[\exp(\lambda T) - 1\right] + \frac{Ci\alpha}{\lambda^2}\left[\exp(\lambda T) - \lambda T - 1\right] \tag{4}$$

Therefore, the total cost per unit time without temporary price discount is

$$TC(T) = \frac{1}{T}\left[A + \frac{C\alpha}{\lambda}\left[\exp(\lambda T) - 1\right] + \frac{Ci\alpha}{\lambda^2}\left[\exp(\lambda T) - \lambda T - 1\right]\right] \tag{5}$$

The convexity of *TC(T)* can be proved similarly as given in Dye et al. (2007). It guarantees that there exists unique value of *T* (say) T^* that minimizes *TC(T)*. T^* can be obtained by setting

$$\frac{dTC(T)}{dT} = A - \frac{C(i+\lambda)\alpha}{\lambda^2}\left[\lambda T\exp(\lambda T) - \exp(\lambda T) + 1\right] = 0 \tag{6}$$

Knowing the regular order cycle time T^*, the optimal order quantity without a temporary price discount, Q^* can be obtained as

$$Q^* = \frac{\alpha}{\lambda}\left[\exp(\lambda T^*) - 1\right] \tag{7}$$

The following two scenarios may arise when the supplier offers a temporary price discount and the retailer take advantage of this price reduction scheme by ordering quantity greater than Q^* : (1) when the special order time occurs at the retailer's cycle time; and (2) when the special order time occurs during the retailer's cycle time. We compute the corresponding total cost savings for these two scenarios.

Scenario 1 : When the special order time occurs at the retailer's cycle time (Figure 1)

Here, the retailer gives special order of Q_s – units at the discounted price. Arguing as above, the inventory level of these Q_s – units at any instant of time *t* is given by

$$I_s(t) = \frac{\alpha}{\lambda}\left[\exp(\lambda(T_s - t)) - 1\right], \; 0 \le t \le T_s \tag{8}$$

and the special order quantity is

$$Q_s = I_s(0) = \frac{\alpha}{\lambda}\left[\exp(\lambda T_s) - 1\right] \qquad (9)$$

For each price discount rate d_i, the total cost $TCS_{1i}(T_s)$ of the special order during the time interval $[0, T_s]$ is

$$TCS_{1i}(T_s) = A + \frac{C(1-d_i)\alpha}{\lambda}\left[\exp(\lambda T_s) - 1\right] + \frac{C(1-d_i)i\alpha}{\lambda^2}\left[\exp(\lambda T_s) - \lambda T_s - 1\right],\ i=1, 2, \ldots, n \qquad (10)$$

On the other hand, if the retailer follows regular order policy of Q^* - units, total cost during $[0, T_s]$ is

$$TCN_1(T_s) = \frac{T_s}{T^*}\left[A + \frac{C\alpha}{\lambda}\left[\exp(\lambda T^*) - 1\right] + \frac{Ci\alpha}{\lambda^2}\left[\exp(\lambda T^*) - \lambda T^* - 1\right]\right] \qquad (11)$$

Clearly, $TCN_1(T_s) > TCS_{1i}(T_s)$ for given d_i. Hence, the total cost savings, $G_{1i}(T_s)$ because of the offer of a temporary price discount is

$$G_{1i}(T_s) = TCN_1(T_s) - TCS_{1i}(T_s) \qquad (12)$$

Scenario 2 : When the special order time occurs during the retailer's cycle time **(Figure 2)**

Here, we study the situation when the special order is to be given during the retailer's cycle time. At this time, the retailer has q – units and orders for Q_s – units which raises his inventory to $W = Q_s + q$. When the special order of Q_s - units is placed, the total cost during the interval $[0, T_W]$ comprises of ordering cost; A, purchase cost as $\frac{C(1-d_i)\alpha}{\lambda}\left[\exp(\lambda T_s) - 1\right]$ and the holding cost which is calculated as follows:

With the arrival of special order quantities, the stock on hand raises from q to W, where

$$W = Q_s + q = \frac{\alpha}{\lambda}\left[\exp(\lambda T_s) - 1\right] + \frac{\alpha}{\lambda}\left[\exp(\lambda t_q) - 1\right] = \frac{\alpha}{\lambda}\left[\exp(\lambda T_s) + \exp(\lambda t_q) - 2\right] \qquad (13)$$

The inventory level at any instant of time t is given by

$$I_W(t) = \frac{\alpha}{\lambda}\left[\exp(\lambda(T_W - t)) - 1\right],\ 0 \le t \le T_W \qquad (14)$$

and $W = I_W(0) = \frac{\alpha}{\lambda}\left[\exp(\lambda T_W) - 1\right]$ (15)

From (13) and (15), we have

$$T_W = \frac{1}{\lambda}\ln\left[\exp(\lambda T_s) + \exp(\lambda t_q) - 1\right] \qquad (16)$$

The holding cost of q – units purchased at \$ C per unit is $\frac{Ci\alpha}{\lambda^2}\left[\exp(\lambda t_q) - \lambda t_q - 1\right]$ and that of the special order quantity Q_s available at \$ $C(1-d_i)$ per unit is

$$C(1-d_i)i\left[\int_0^{T_W} I_W(t)dt - \frac{\alpha}{\lambda^2}\left[\exp(\lambda t_q) - \lambda t_q - 1\right]\right]$$

$$= \frac{C(1-d_i)i\alpha}{\lambda^2}\left[\exp(\lambda T_s) + \exp(\lambda t_q) - 2 - \ln(\exp(\lambda T_s) + \exp(\lambda t_q) - 1) - (\exp(\lambda t_q) - \lambda t_q - 1)\right] \qquad (17)$$

Therefore, the total holding cost of W – units during $[0, T_W]$ is

$$\frac{C(1-d_i)i\alpha}{\lambda^2}\left[\exp(\lambda T_s) + \exp(\lambda t_q) - 2 - \ln(\exp(\lambda T_s) + \exp(\lambda t_q) - 1)\right] + \frac{Cid_i\alpha}{\lambda^2}(\exp(\lambda t_q) - \lambda t_q - 1)$$

(18)

Hence, for the fixed price discount rate d_i, the total cost $TCS_{2i}(T_s)$ of the special order during $[0, T_W]$ is

$$TCS_{2i}(T_s) = A + \frac{C(1-d_i)\alpha}{\lambda}\left[\exp(\lambda T_s) - 1\right] + \frac{Cid_i\alpha}{\lambda^2}(\exp(\lambda t_q) - \lambda t_q - 1) + \frac{C(1-d_i)i\alpha}{\lambda^2}\left[\exp(\lambda T_s) + \exp(\lambda t_q) - 2 - \ln(\exp(\lambda T_s) + \exp(\lambda t_q) - 1)\right] \quad (19)$$

If the retailer follows regular order policy, the total cost during the interval $[0, T_W]$ will be computed for two periods. In the first period, he incurs the holding cost for q – units as $Ci\int_0^{t_q} I(t)dt = \frac{Ci\alpha}{\lambda^2}\left[\exp(\lambda t_q) - \lambda t_q - 1\right]$ and in the next period total cost as

$$\frac{(T_W - t_q)}{T^*}\left[A + \frac{C\alpha}{\lambda}\left[\exp(\lambda T^*) - 1\right] + \frac{Ci\alpha}{\lambda^2}\left[\exp(\lambda T^*) - \lambda T^* - 1\right]\right]$$
$$= \frac{(\ln(\exp(\lambda T_s) + \exp(\lambda t_q) - 1) - \lambda t_q)}{\lambda T^*}\left[A + \frac{C\alpha}{\lambda}\left[\exp(\lambda T^*) - 1\right] + \frac{Ci\alpha}{\lambda^2}\left[\exp(\lambda T^*) - \lambda T^* - 1\right]\right]$$

Therefore, the total cost of the inventory system is

$$TCN_2(T_s) = \frac{Ci\alpha}{\lambda^2}\left[\exp(\lambda t_q) - \lambda t_q - 1\right] + \frac{(\ln(\exp(\lambda T_s) + \exp(\lambda t_q) - 1) - \lambda t_q)}{\lambda T^*}\left[A + \frac{C\alpha}{\lambda}\left[\exp(\lambda T^*) - 1\right] + \frac{Ci\alpha}{\lambda^2}\left[\exp(\lambda T^*) - \lambda T^* - 1\right]\right] \quad (20)$$

From (19) and (20), for a fixed discount rate d_i, the total cost savings; $G_{2i}(T_s)$ is

$$G_{2i}(T_s) = TCN_2(T_s) - TCS_{2i}(T_s) \quad (21)$$

Obviously, the total cost savings in (12) and (21) have to be positive to qualify for special order quantity.

ANALYTIC RESULTS

In this section, the optimal value of T_s is calculated that maximizes the total cost savings.

Scenario 1 : When the special order time occurs at the retailer's cycle time

For the fixed discount rate d_i, the derivative of $G_{1i}(T_s)$ in (12) with respect to T_s gives

$$\frac{dG_{1i}(T_s)}{dT_s} = \frac{1}{T^*}\left[A + \frac{C\alpha}{\lambda}\left[\exp(\lambda T^*) - 1\right] + \frac{Ci\alpha}{\lambda^2}\left[\exp(\lambda T^*) - \lambda T^* - 1\right]\right] - C(1-d_i)\alpha\exp(\lambda T_s) - \frac{C(1-d_i)i\alpha}{\lambda^2}\left[\exp(\lambda T_s) - 1\right] \quad (22)$$

and $$\frac{d^2G_{1i}(T_s)}{dT_s^2} = -C(1-d_i)(\lambda + i)\alpha\exp(\lambda T_s) < 0 \quad (23)$$

Eq. (23) proves that $G_{1i}(T_s)$ is a concave function of T_s. Hence, a unique value of $T_s = T_{s1i}$ (say) exists that maximizes $G_{1i}(T_s)$. Equating (22) to be zero gives value of T_{s1i} as

$$T_{s1i} = \frac{1}{\lambda} \ln \left[\frac{C(1-d_i)i\alpha + \lambda x}{C(1-d_i)\alpha(\lambda+i)} \right] \tag{24}$$

where $x = \frac{1}{T^*}\left[A + \frac{C\alpha}{\lambda}\left[\exp(\lambda T^*) - 1\right] + \frac{Ci\alpha}{\lambda^2}\left[\exp(\lambda T^*) - \lambda T^* - 1\right]\right] > 0$.

Clearly, $Q^* < Q_{s1i}$ if and only if $T^* < T_{s1i}$ .i.e. $\Delta_{1i} > 0$ (25)

where $\Delta_{1i} = x - C(1-d_i)\alpha \exp(\lambda T^*) - \frac{C(1-d_i)i\alpha}{\lambda^2}\left[\exp(\lambda T^*) - 1\right]$

Using (24) in (12) gives the corresponding maximum total cost savings as

$$G_{1i}(T_{s1i}) = \frac{C(1-d_i)\alpha(\lambda+i)}{\lambda^2}\left[\lambda T_{s1i}\exp(\lambda T_{s1i}) - \exp\left(\lambda T_{s1i}\right) + 1\right] - A \tag{26}$$

Denote $\Delta_{2i} = G_{1i}(T_{s1i})$. The retailer orders special order only if $\Delta_{2i} > 0$. Otherwise, he will continue with the regular order policy of Q^*- units. Hence, the optimal value of T_{s1i} (denoted by $T_{s1i}{}^*$) for scenario 1 is

$$T_{s1i}{}^* = \begin{cases} T_{s1i}, & \text{if } \Delta_{1i} > 0 \text{ and } \Delta_{2i} > 0 \\ T^*, & \text{otherwise} \end{cases} \tag{27}$$

Scenario 2 : When the special order time occurs during the retailer's cycle time

For the fixed price discount rate d_i, the first order condition for maximizing $G_{2i}(T_s)$ in (21) with respect to T_s

$$\frac{dG_{2i}(T_s)}{dT_s} = \left[x + \frac{C(1-d_i)i\alpha}{\lambda}\right]\frac{\exp(\lambda T_s)}{\exp(\lambda T_s) + \exp(\lambda t_q) - 1} - \frac{C(1-d_i)(\lambda+i)\alpha}{\lambda}\exp(\lambda T_s) = 0 \tag{28}$$

gives $T_s = T_{s2i}$ (say)

$$T_{s2i} = \frac{1}{\lambda}\ln\left[\frac{\lambda x + C(1-d_i)i\alpha - C(1-d_i)(\lambda+i)\alpha\exp(\lambda T_s)}{C(1-d_i)(\lambda+i)\alpha}\right] \tag{29}$$

The second order derivative

$$\left.\frac{d^2G_{2i}(T_s)}{dT_s{}^2}\right|_{T_s = T_{s2i}} = -\frac{C(1-d_i)(\lambda+i)\alpha}{\exp(\lambda T_s) + \exp(\lambda t_q) - 1}\exp(2\lambda T_{s2i}) < 0$$

suggests that $G_{2i}(T_{s2i})$ is maximum. Next to ensure that $Q^* < Q_{s2i}$ i.e. $T^* < T_{s2i}$, substitute (29) into this inequality which results in $T^* < T_{s2i}$ if and only if $\Delta_{3i} > 0$ (30)

where $\Delta_{3i} = x - \frac{C(1-d_i)(\lambda+i)\alpha}{\lambda}\left(\exp(\lambda T^*) + \exp(\lambda t_q) - 1\right) + \frac{C(1-d_i)i\alpha}{\lambda}$.

Using (29) into (21) gives the corresponding maximum total cost savings as

$$G_{2i}(T_{s2i}) = \frac{1}{\lambda^2}C(1-d_i)(\lambda+i)\alpha\left[\ln(\exp(\lambda T_{s2i}) + \exp(\lambda t_q) - 1) - \lambda t_q\right](\exp(\lambda T_{s2i}) + \exp(\lambda t_q) - 1)$$
$$-\frac{1}{\lambda^2}C(1-d_i)(\lambda+i)\alpha(\exp(\lambda T_{s2i}) - 1) - A \tag{31}$$

Denote $G_{2i}(T_{s2i})$ by Δ_{4i}. To qualify for special order quantity $\Delta_{4i} > 0$ otherwise the retailer is advised to opt for the regular order policy. Hence, the optimal value of T_{s2i} (denoted by T_{s2i}^*) for scenario 2 is

$$T_{s2i}^* = \begin{cases} T_{s2i}, & \text{if } \Delta_{3i} > 0 \text{ and } \Delta_{4i} > 0 \\ T^*, & \text{otherwise} \end{cases} \tag{32}$$

The following computational procedure is outlined to calculate the optimal special order cycle time T_s^* and the optimal special order quantity Q_s^* for both the scenarios.

Computational Procedure

Step 1. If $q = 0$, then compute T^* and go to step 2. Otherwise calculate t_q from $t_q = \frac{1}{\lambda}\ln\left(1+\frac{\lambda q}{\alpha}\right)$, and go to step 4.

Step 2. For each d_i, $i = 1, 2, \ldots, n$ obtain T_{s1i} from (24), Δ_{1i} from (25) and Δ_{2i} from (26). If $\Delta_{1i} > 0$ and $\Delta_{2i} > 0$ then substitute T_{s1i} into (9) and obtain Q_{s1i}. Check Q_{s1i} under di. If

(i) $Q_i \le Q_{s1i} < Q_{i+1}$, then Q_{s1i} is a feasible solution. Set $Q_{s1i}^* = Q_{s1i}$ and compute $G_{1i}(T_{s1i}^*)$.

(ii) $Q_{s1i} \ge Q_{i+1}$, then larger price discount rate is possible and thus Q_{s1i} is not a feasible solution. Set $G_{1i}(T_{s1i}^*) = -\infty$.

(iii) $Q_{s1i} < Q_i$ then set $Q_{s1i}^* = Q_i$. Substitute Q_{s1i}^* into (9) and find T_{s1i}^* and hence compute $G_{1i}(T_{s1i}^*)$. If $G_{1i}(T_{s1i}^*) > 0$, go to step 3; otherwise set $T_{s1i}^* = T^*$, $Q_{s1i}^* = Q^*$ and $G_{1i}(T_{s1i}^*) = 0$.

Step 3. Find $\max_{i=1,2,\ldots,n} G_{1i}(T_{s1i}^*)$. Go to step 6.

Step 4. For each d_i, $i = 1, 2, \ldots, n$ obtain T_{s2i} from (29), Δ_{3i} and Δ_{4i}. If $\Delta_{3i} > 0$ and $\Delta_{4i} > 0$ then substitute T_{s2i} into (13) and obtain Q_{s2i}. Check Q_{s2i} under di. If

(i) $Q_i \le Q_{s2i} < Q_{i+1}$, then Q_{s2i} is a feasible solution. Set $Q_{s2i}^* = Q_{s2i}$ and compute $G_{2i}(T_{s2i}^*)$.

(ii) $Q_{s2i} \ge Q_{i+1}$, then larger price discount rate is possible and thus Q_{s2i} is not a feasible solution. Set $G_{2i}(T_{s2i}^*) = -\infty$.

(iii) $Q_{s2i} < Q_i$ then set $Q_{s2i}^* = Q_i$. Substitute Q_{s2i}^* into (9) and find T_{s2i}^* and hence compute $G_{2i}(T_{s2i}^*)$. If $G_{2i}(T_{s2i}^*) > 0$, go to step 5; otherwise set $T_{s2i}^* = 0$, $Q_{s2i}^* = 0$ and $G_{2i}(T_{s2i}^*) = 0$.

Step 5. Find $\max_{i=1,2,\ldots,n} G_{2i}(T_{s2i}^*)$. Go to step 6.

Step 6. Stop.

In the next section, numerical examples are considered to support the proposed problem.

NUMERICAL EXAMPLES

Example 1 Consider the following parametric values for the retailer inventory system when the special order is due at the regular order cycle time : C = \$ 10 / unit, α = 1000 units / year, A = \$ 150 / order, i = 30 % per annum, β = 10 %. Using step 1 of computational procedure, the optimum cycle time T^* = 0.271 years and regular order quantity is Q^* is 275 units per order. The price discount rate offered by the supplier is given in Table 1.

Table 1. Price discount rate schedule

Class I	Special order quantity Q_s	Discount rate d_i
1	$500 \le Q_s < 1000$	10 %
2	$1000 \le Q_s < 2400$	20 %
3	$Q_s \ge 2400$	28 %

Using steps 2 and 3, the solution is obtained as given in Table 2.

Table 2. Optimal solutions for Example 1

d_i	Q_{s1i}	T_{s1i}^*	Q_{s1i}^*	G_{1i}^*
10 %	583	0.567	583	451.17
15 %	765	0.953	1000	733.94
28 %	969	2.151	2400	1072.40

Shaded solution is the optimal solution.

From Table 2, it is observed that the retailer saves \$ 1072.40 by ordering 2400 – units available at the discount rate 28 %.

Example 2. Consider the data as given in example 1 except for q. Here, we want to validate scenario 2 when the special order time is during the retailer's cycle time. The optimal ordering policies for q = 50, 100 and 200 are given in Table 3.

Table 3. Optimal solutions for Example 2 for different values of q

q	T_{s2}^*	Q_{s2}^*	G_2^*
50	2.151	583	757.37
100	2.151	1000	445.16
200	0.953	2400	80.95

From Table 3, it is can be seen that the total cost savings is negatively very sensitive to the remnant inventory. It directs the logistic manager to keep remnant inventory as low as possible when the special order time occurs during the cycle time.

Example 3. In Table 4, we study the effect of changes in the inventory parameters C, α, A, i and β on the optimal price discount rate, special order quantity and total cost savings. The data is taken as that of Example 2 and q = 50.

Parameter	Value	d_i^*	Q_{s2}^*	G_2^*
C	5.0	0.28	2400	1073.76
	7.5	0.28	2400	945.51
	12.5	0.28	2400	714.65
	15.0	0.28	2400	560.74
α	500	0.10	500	474.75
	750	0.20	1000	665.71
	1250	0.28	2400	792.29
	1500	0.28	2400	1024.65
A	25.0	0.20	1000	750.61
	37.5	0.20	1000	792.75
	112.5	0.28	2400	1284.68
	225.0	0.28	2400	1482.39
i	0.15	0.28	2400	3258.18
	0.25	0.28	2400	1630.89
	0.35	0.20	1000	1018.17
	0.45	0.20	1000	710.47
β	0.05	0.20	1000	518.41
	0.15	0.28	2400	892.04
	0.20	0.28	2400	915.16
	0.25	0.28	2400	962.80

From Table 4, the retailer has following observations :

(1) The optimal special order quantity is determining by comparing available price discount to additional holding cost which he will incur. For example, for α = 500 or β = 5 %, the retailor follows the regular order policy. The retailer maximizes his total cost savings by selecting the appropriate price discount rate.

(2) When demand; α or ordering cost: A, are likely to increase, it is advantageous to stock more at a discounted price.

(3) Increase in holding charge fraction decreases special order quantity and total cost savings.

(4) Increase in the stock-dependent parameter increases the decision variables resulting more saving in the total cost.

(5) Increase in the deterioration rate decreases the special order quantity and total cost savings. So retailer should have modern facilities to stock items which are subject to deterioration.

CONCLUSION

The optimal ordering policy for a retailer is suggested when a supplier offers a temporary price discount if larger order is placed compared to regular one. The demand is stock-dependent and units in inventory are subject to constant deterioration. The optimal policy of special order is determined to maximize the total cost savings. A computational procedure is outlined to determine the appropriate optimal policy. Numerical examples are given to support the theoretical results. A sensitivity analysis is carried out to study the effect of changes in the inventory parameters on the optimal solution. It is advised to the retailer to keep his remnant inventory as low as possible because the contributed holding cost of the remnant inventory is very high. The retailer should opt for the special order quantity when the unit price, market demand and ordering cost are likely to increase. This decision policy will help the retailer to sustain in the competitive market when his demand increases because of display of goods in the showroom.

The developed model can be studied to compare various promotional schemes offered by the supplier. This model should be for different demand structures and time dependent deterioration. The effect of inflation can also be incorporated.