A High School First Course in

EUCLIDEAN PLANE

GEOMETRY

Charles H. Aboughantous
The shape, position and order are not basic but are born in the imagination

Democritus
INTRODUCTION

The book is designed to promote the art and the skills of developing logical proofs of geometry propositions. It is concise, to the point and is presented to form a first course of geometry at high school level.

The content of the book is based on Euclid’s five postulates and the most common theorems of plane geometry. Some of the theorems are introduced with detailed proofs. Other theorems are introduced because of their usefulness but their proofs are left as challenging problems to the users.

Problem solving examples are scattered throughout the book. They illustrate the organization of the proofs’ written statements. Numerous annotations are introduced in the statements of the solutions to help the student understand the justification of the steps in the progress of the solution. The Author recommends that the student reproduce the solutions on his own to maximize the benefit from the book.

The content of the book is introduced in eleven chapters. The first chapter presents the five Euclid’s postulates of plane geometry. The other chapters are organized in groups of subjects: the line, angles, triangles, quadrilaterals, similarity, circle, and elements of space geometry. A last chapter introduces strategies in calculating surface areas and volumes of non-simple objects. Those are mini engineering problems.

Each chapter is appended by a set of Practice Problems that could be worked out as class assignments or just for practice. Those problems are presented in three groups: construction problems, computational problems, and theoretical problems.

The first group helps the student to develop dexterity of drawing geometric figures using standard drawing instruments, a ruler, a protractor and a compass. The second group requires basic algebra skills. The student computes parts of geometric figures using data of other parts and pertinent theorems. Many of those problems are simplified ver-
isions of real engineering problems. They pave the way to workout the problems of the last chapters. The last group is where the student sharpens his talent of developing logical proofs.

Although the book is intended to be on plane geometry, the chapter on space geometry seems unavoidable. It helps understanding the figures and the shapes of solid objects. The surface area of solid objects can be computed using plane geometry formulae upon converting the surface of a solid to a plane surface. The portion of space comprised within the surface of a solid is a volume. Methods of calculating volumes of non-simple solids are introduced in that chapter.

The book has two companions, the Solutions Manual and the Power Point package. The users will have to acquire those separately. The Solutions Manual contains the detailed solutions to all the problems in the book. The Power Point package (incomplete at the time of printing this volume) contains animated and annotated presentations of geometric constructions. Some of those are described in the textbook; the others are answers to the construction problems presented at the end of each chapter of the book.

My thanks go to Mabel Castleberry for her generous time and efforts editing the manuscript.

Charles Aboughantous
votan@sprintmail.com
This prologue is intended to share with the users my experience in teaching geometry using this book. The students who followed my recommendations about how to study with this book achieved a stunning improvement in their grades as well as in their initiatives in solving problems. This is how it worked with my students.

I require that all the students in my class have a binder dedicated exclusively to the geometry course and use the paper from that binder for the homework and for the quizzes. That way they will be able to re-file the graded papers in the same folder for future references. Also, I ask that the students use the binder to take notes of everything I write on the board. Most of them do!

In a typical new lesson I write the statement of a theorem and I draw the figure on the board. I carry out the proof and I ask the students to do the same on their binders; alternatively I start with a problem from the book. Then I present a new problem. I draw a figure on the board and I ask the students to prove something about it before I carry out the solution on the board. The new question could be to prove the theorem itself, but now using a different figure. The new figure could be the one I used in the proof of the theorem but now it is rotated or stretched and with different letters.

Sometimes I add lines to the new figure that are not relevant to the question. In those instances I instruct the students to focus on the parts of the figure that are relevant to the question and to ignore the non-relevant parts. I show them how to strip the extraneous information from the figure using problems from the book. An extreme example to this approach is illustrated in problem 26 of Chapter 6 in the Solutions Manual.

In some problems breaking up the figure into simpler ones is not an option. I highlight the parts that would make it easier for the students to see what in the figure is relevant to the question. A typical example to this approach is illustrated in problem 8 of Chapter 4 in the Solutions Manual.
The first requirement to do well in any course is to study. Almost everyday I assign a part from the book to be a quiz for the next day. The assignment is typically a theorem and its proof; alternatively the assignment could be a problem from the book. I instruct the students how to study; sometimes I ask the parents who care to help their children to manage the studying time at home and to make sure their children follow my instructions of studying.

I recommend that the students copy the assigned study theorem and its proof three times. I recommend not looking at the figure of the book while copying the theorem and the proof. I instruct the students to copy the figure on their papers. It is important to imprint the image of the figure and the location of the letters in the figure in the mind of the students. The letters of the figures identify the parts of the logical development of the proof. I encourage the students to use different letters from the ones used in the figure of the book.

I instruct the students to focus on memorizing the statement of the theorems as written; they will be able to state the same using their own language later. Also, I ask the students to memorize the steps and the structure of the proofs of the theorems. That’s not harmful for a start. With practice on solving problems guided by the logical structure of the proofs they have studied, they will be able to develop their own logical proofs later. This has proven particularly true in problems that could be proven with different methods using different theorems. The memorization here is comparable to the memorization of the best move for a given disposition of the pieces on the chess game board.

After copying the assignment three times from the book, I recommend the students close the book and to try to redo the proof out of their memory, by first sketching the figure then writing the proof. After completing this rehearsal they should compare their proofs with that of the book. I recommend that the student fix their errors by writing them down on that last attempt of writing their proofs.
I ask the students to write the theorem and the proof without looking at the book for the last time, just before they go to bed. Again fix the errors by writing them on the paper of their solutions. By completing this task they should be ready for a successful next day quiz. It didn’t take too long with my students before they begin to develop their own proofs without that lengthy rehearsal.

I instruct the students to do freehand sketches of the figures as closely as possible to the figure of the book. Most of my students had a hard time performing this task early in the course. Most of them improved on that deficiency. I do freehand sketches on the board and I ask the students to follow me step by step by drawing one line at a time on their papers with me. I rarely use drawing instruments except in geometric construction problems and I ask the students to follow my example. Most of them do!

I always use colors in sketching figures on the board. I observed that most students find it easier to distinguish the different parts of the colored figures. That helped improve their understanding of the question, which is essential for succeeding in carrying out the solution. Methods of solving problems and various alternatives to name angles and analyzing figures in problem solving are presented with ample details in the Solutions Manual.

I encourage the users to be creative.
Greek alphabet
Highlighted lower case letters are used for angles

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>Alpha</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>β</td>
<td>Beta</td>
<td>$\beta$</td>
</tr>
<tr>
<td>γ</td>
<td>Gamma</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>δ</td>
<td>Delta</td>
<td>$\delta$</td>
</tr>
<tr>
<td>ε</td>
<td>Epsilon</td>
<td>$\epsilon$</td>
</tr>
<tr>
<td>ζ</td>
<td>Dzeta</td>
<td>$\zeta$</td>
</tr>
<tr>
<td>η</td>
<td>Eta</td>
<td>$\eta$</td>
</tr>
<tr>
<td>θ</td>
<td>Theta</td>
<td>$\theta$</td>
</tr>
<tr>
<td>ϰ</td>
<td>Iota</td>
<td>$\iota$</td>
</tr>
<tr>
<td>κ</td>
<td>Kappa</td>
<td>$\kappa$</td>
</tr>
<tr>
<td>λ</td>
<td>Lambda</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>μ</td>
<td>Mu</td>
<td>$\mu$</td>
</tr>
<tr>
<td>ν</td>
<td>Nu</td>
<td>$\nu$</td>
</tr>
<tr>
<td>ξ</td>
<td>Ksi</td>
<td>$\xi$</td>
</tr>
<tr>
<td>ο</td>
<td>Omicron</td>
<td>$\omicron$</td>
</tr>
<tr>
<td>π</td>
<td>Pi</td>
<td>$\pi$</td>
</tr>
<tr>
<td>ρ</td>
<td>Rho</td>
<td>$\rho$</td>
</tr>
<tr>
<td>σ</td>
<td>Sigma</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>τ</td>
<td>Tau</td>
<td>$\tau$</td>
</tr>
<tr>
<td>υ</td>
<td>Upsilon</td>
<td>$\upsilon$</td>
</tr>
<tr>
<td>φ</td>
<td>Phi</td>
<td>$\phi$</td>
</tr>
<tr>
<td>χ</td>
<td>Chi</td>
<td>$\chi$</td>
</tr>
<tr>
<td>ψ</td>
<td>Psi</td>
<td>$\psi$</td>
</tr>
<tr>
<td>ω</td>
<td>Omega</td>
<td>$\omega$</td>
</tr>
</tbody>
</table>

Mathematical symbols used in this book
They are also defined at the first occurrence in the book

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\parallel$</td>
<td>Parallel</td>
<td>Line $AB$ parallel to line $CD$: $AB \parallel CD$</td>
</tr>
<tr>
<td>$\perp$</td>
<td>Perpendicular</td>
<td>Line $AB$ perpendicular to line $CD$: $AB \perp CD$</td>
</tr>
<tr>
<td>$\cong$</td>
<td>Congruence</td>
<td>Object $A$ congruent to object $B$: $A \cong B$</td>
</tr>
</tbody>
</table>
| $\neg$ | Negation   | Object $A$ is not object $B$: $A \neg B$
Read as: $A$ not $B$. |
| AND | Boolean logical operators | Association: both $A$ and $B$ together: $A \land B$
Alternate: either $A$ or $B$: $A \lor B$
Negation: one not the other: $A \neg B$ |
| $\Rightarrow$ | Implication | Also used in the book for therefore |
| $\equiv$ | Identity, definition | $A$ is identical to $A$;
also $A$ is define as $B$: $A \equiv B$ |
| $\exists$ | Existence | There exists an object $Q$: $\exists Q$ |
| $\cap$ | Intersection | Line $m$ intersects with line $k$ at point $P$:
$P = m \cap k$
Read as: $P$ is $m$ inter $k$; also $P$ is defined as |
| $\in$ | Belongs to | $A \in k$ : $A$ is a point of line $k$; also $A$ belongs to $k$,
$A$ is contained in $k$. |
| $\angle A$ | Angle | $A$ is the vertex of the angle |
| $\overline{BAD}$ | Angle | $A$ is the vertex of the angle, $BA$ and $AD$ are its sides:
read as angle $BAD$. |
| $\overline{RB}$ | Arc | $R$ and $B$ are the endpoints of the arc |
| $\overline{ABC}$ | Arc | $A$ and $C$ are the endpoints of the arc, $B$ is any point on the arc. |
| $\propto$ | Proportion | $a$ is proportional to $b$: $a \propto b$ |
| $\sim$ | Similarity | $A$ is similar to $B$: $A \sim B$ |

Annotations: annotations with an arrow, such as $\leftarrow$ use supplementary angles, are inserted in the solutions of the examples in the body of the chapters to focus the attention of the student to what was done at that point of the solution. The student does not need to add those comments to his work.

An Asterisk (*) preceding the statement of the problem in the Practice Problems sections means there is a figure associated with the problem on that page.
# Table of Contents

## Chapter One
The Postulates Of Plane Geometry

I. Euclid’s postulates ............................................................... 1  
   Postulate 1 ................................................................. 1  
   Postulate 2 ................................................................. 2  
   Postulate 3 ................................................................. 2  
   Postulate 4 ................................................................. 3  
   Postulate 5 ................................................................. 3  
   Types of lines ............................................................. 3  
   The unit of length ......................................................... 3  

II. Geometric Construction of segments ........................................ 4  
   Construction of segment of a given length .......................... 4  
      Compass method ................................................... 4  
      Ruler method ....................................................... 4  
   Construction of the midpoint of segment AB ......................... 5  
      Compass method ................................................... 5  
      Ruler method ....................................................... 5  
   Construction of a line at right angle with another line .......... 6  
   Construction of a line parallel to a given line .................... 6  

Practice Problems ............................................................ 7

## Chapter Two
The Straight Line

I. Binary operations ............................................................. 8  
   Arithmetic operations .................................................. 8  
   Geometric operations ................................................... 9  
      Addition of two parallel segments .............................. 9  
      Addition of two non-parallel segments ......................... 10  
   Logical operations ..................................................... 10  
      Boolean operators .................................................. 10  
      Whole object operators .......................................... 11  
      Size operators ..................................................... 12
CHAPTER THREE
Angles One: Shapes and measures

I. Types of angles ................................................................. 17
   Labeling angles ............................................................. 18
   Adjacent angles ............................................................. 19

II. Measuring angles ............................................................ 20
   Binary operations of angles ............................................... 21

III. Construction of angles
   Given line \( m \) construct line \( k \perp m \) .................................. 23
   Construct the bisector of an angle
   Compass method .............................................................. 24
   Protractor method ........................................................... 24
   Construct an angle congruent to a given angle
   Compass method .............................................................. 24
   Construct an angle its measure \( \alpha ^\circ \) is known ...................... 25
   Construct the angle sum and the angle difference of two angles
   Angle sum ................................................................. 25
   Angle difference ............................................................. 25

Practice problems ............................................................... 26

CHAPTER FOUR
Angles Two: Basic theorems

Theorem 3. ................................................................. 28
Theorem 4. ................................................................. 28
Reciprocal 4. ............................................................... 29
   Opposite angles (also vertical angles) .............................. 29
Theorem 5. ................................................................. 29
Theorem 6. ................................................................. 29
CHAPTER FIVE
Triangles One: Basic theorems

I. Definitions ......................................................... 35

II. Properties of triangles ........................................... 36
   Nomenclature of triangles .................................... 37
   Metric properties of triangles ............................... 38

III. Construction of triangles
   Construct a triangle its sides are 6 cm, 5 cm and 3 cm .......... 39
   Construct a triangle its base is 6 cm adjacent to two angles 45° and 30° ........................................... 39
   Construct a triangle its sides are 6 cm and 4 cm making an angle of 45° .............................................. 40
   Construct a triangle having one angle of 30°, one side adjacent to 30° is 4 cm long, and the side opposite to 30° is 3 cm long ............................................. 40

IV. Basic theorems of triangles
   Theorem 12. ....................................................... 40
   Theorem 13. ....................................................... 41
   Theorem 14. ....................................................... 41
   Theorem 15. ....................................................... 42
   Reciprocal 15. ................................................... 42
   Theorem 16. ....................................................... 42
   Theorem 17. ....................................................... 42
Chapter Six
Triangles Two: Congruence theorems

I. Introduction ................................................................. 51

II. Congruence theorems

SAS theorem
Theorem 23. ................................................................. 51

ASA theorem
Theorem 24. ................................................................. 52

SSS theorem
Theorem 25. ................................................................. 53

HA theorem
Theorem 26. ................................................................. 53

HS theorem
Theorem 27. ................................................................. 54

Practice problems .......................................................... 55

Chapter Seven
Quadrilaterals And regular polygons

I. Introduction
Simple quadrilateral .................................................. 59
Parallelogram .............................................................. 59
## Metric properties of parallelograms

- **Rhombus**

## Rhombus

**Metric properties of rhombus**

## Rectangle

**Metric properties of rectangles**

## Square

**Metric properties of squares**

## Trapezoid

**Metric properties of trapezoids**

### II. Construction of quadrilaterals

**Construction of a rectangle**

**Construction of a parallelogram**

**Construction of a trapezoid**

### III. Basic quadrilateral theorems

- **Theorem 28.**
- **Theorem 29.**
- **Theorem 30.**
- **Reciprocal 30.**
- **Theorem 31.**
- **Theorem 32.**
- **Reciprocal 32.**
- **Theorem 33.**
- **Theorem 34.**
- **Theorem 35.**
- **Reciprocal 35.**
- **Theorem 36.**
- **Theorem 37.**
- **Theorem 38.**
- **Reciprocal 38.**
- **Theorem 39.**

### IV. Properties of regular polygons

- **A polygon** (defined)
- **Nomenclature of polygons** (Table 1)
- **Metric properties of regular polygons of n sides**
- **Symmetry properties of polygons of n sides**

### V. Constructions of polygons

- **Construct a hexagon**
Construct an octagon .................................................. 70
Practice problems ....................................................... 71

Chapter Eight
Similitude

I. Introduction .............................................................. 74
The similitude ratio ...................................................... 75
Cross product ............................................................. 76

II. Basic similitude theorems

Thales Theorem
Theorem 40. ............................................................... 78
Reciprocal 40. ............................................................ 79
Theorem 41. ............................................................... 79

The proportion theorem
Theorem 42. ............................................................... 79

AA theorem
Theorem 43. ............................................................... 80

SSS similitude theorem
Theorem 44. ............................................................... 80

SAS similitude theorem
Theorem 45. ............................................................... 80
Theorem 46. ............................................................... 80
Theorem 47. ............................................................... 81
Theorem 48. ............................................................... 81
Theorem 49. ............................................................... 83
Theorem 50. ............................................................... 83

Similitude of polygons ................................................. 83
Theorem 51. ............................................................... 84
Theorem 52. ............................................................... 84

III. Application of similitude
Partition a line segment into congruent segments .......... 84
Enlarge a given triangle ............................................. 85
Chapter Nine
The Circle

I. Introduction ................................................................. 89

   Metric properties of circles ........................................... 90

II. Properties of circles .................................................. 91

   Intersection of lines and circles with circles ..................... 91
   Construct a tangent to a circle ..................................... 91
   Circles share common tangent lines ............................... 92
   Inscribed and circumscribing circles ............................... 92

III. Basic theorems of circles
   Theorem 53. ............................................................... 93
   Corollary 53. ............................................................. 94
   Theorem 54. ............................................................... 94
   Theorem 55. ............................................................... 94
   Theorem 56. ............................................................... 94
   Reciprocal 56. ........................................................... 94
   Theorem 57. ............................................................... 94
   Theorem 58. ............................................................... 95
   Theorem 59. ............................................................... 95
   Theorem 60. ............................................................... 97
   Theorem 61. ............................................................... 97
   Theorem 62. ............................................................... 98
   Theorem 63. ............................................................... 98

   Power of a point (defined) ............................................ 97
   Theorem 60. ............................................................... 97
   Theorem 61. ............................................................... 97
   Theorem 62. ............................................................... 98
   Theorem 63. ............................................................... 98

Practice problems .......................................................... 99

Chapter Ten
Basic Elements Of Space Geometry

I. Introduction ................................................................. 104

II. Propositions of space geometry .................................... 105

   Lines and planes in general ........................................ 105
Perpendicular and orthogonal lines ........................................ 106
Intersecting and parallel planes ........................................... 106

III. Construction of figures in planes
Construct a line in a plane .................................................. 106
Construct a line intersecting with a plane ............................. 106
Construct a line perpendicular to a plane .............................. 106
Construct a plane intersecting with another plane ................... 107
Construct a plane perpendicular to another plane .................... 107
Construct two parallel planes .............................................. 107
Construct a line intersecting with two parallel planes .......... 107
Construct a right triangle in a plane ..................................... 107
Construct a rectangle in a plane .......................................... 107
Construct a triangle in $\sigma \perp \pi$ .......................................... 108

IV. Polyhedra ........................................................................ 108
Particular cases of polyhedra are
Ortho-tetrahedron ............................................................. 108
Ortho-dodecahedron .......................................................... 109
Pyramidal polyhedra ............................................................ 109
Metric properties of pyramids ............................................. 110
Frustum .............................................................................. 111
Prismatic polyhedra .............................................................. 112
Conical polyhedra ............................................................... 113
Cylinder ............................................................................ 113
Metric properties of cylinders ............................................ 114
Cone ............................................................................... 114
Metric properties of cones ................................................ 115
Sphere ............................................................................... 116
Metric properties of spheres .............................................. 117

VI. Construction of polyhedral solids
Construction of a tetrahedron .............................................. 118
Construction of a hexahedron .............................................. 119
Construction of an octahedron .......................................... 119
Construction of a dodecahedron ........................................ 119
Construction of an icosahedron ........................................ 120
Construction of a cylinder .................................................. 120
Construction of a cone ....................................................... 121

Practice problems .................................................................. 122
CHAPTER ELEVEN
Methods In Areas And Volumes

I. Methods in planar surface areas ................................................. 125
II. Methods in space objects ........................................................ 129
Practice problems ................................................................. 134

ANSWERS TO COMPUTATIONAL PROBLEMS .......................... 140
Alphabetical Index ............................................................... 142
CHAPTER ONE

THE POSTULATES OF
PLANE GEOMETRY

I. Euclid’s Five Postulates

Geometry is a branch of mathematics that deals with shapes of objects drawn as lines, triangles, circles and boxes, among others. It is based on the definition of a point and some postulates. A postulate is a proposition that is accepted true without proof. If a proposition could be proven then it is not a postulate.

Definitions:

- A point is something that has no part.
- A plane is a flat surface, such as the surface of the board. A plane has no thickness.

We visualize a point by a spot on the plane. A practical plane would be a flat sheet of paper, or the board. The spot contains an infinite number of points. However, for all practical purposes a point is always visualized by marks of small shapes. We usually label a point by an upper case letter, such as point \( A \) (fig. 1). The figures we work with in this course are drawn in a plane, such as the sheet of paper, or the board. For this reason our geometry is said plane geometry.

Euclidean plane geometry is based on five Postulates all of which make use of the definition of a point and a plane.

**Postulate 1.** Only one straight-line segment can be drawn joining any two points.

A straight line segment, or simply a segment that joins two points \( A \) and \( B \) may be visualized by marking the trace of a pencil on the paper with the help of a ruler. A line segment is named by its endpoints, such as segment \( AB \) (fig. 2). We could write \( AB \) or \( BA \). It is the same segment in this book.
We note *en passant* that a *directed* segment is presented by its endpoints letters and a bar atop of the letters, such as $\overline{AB}$, which means: the segment stretches from the starting point $A$ to the end point $B$. Therefore, $\overline{AB}$ and $\overline{BA}$ are two different segments. Directed segments are not covered in this book.

**Postulate 2.** A straight-line segment can be extended indefinitely on either sides in a straight line.

A straight line is always of an infinite length. It is impossible to visualize the entire line. Instead we draw a segment of a straight-line without specifying endpoints. Such a straight line is referred to as a line through points $A$ and $B$ (fig. 3). Most often we do not need to use two points to specify a line. We draw a line segment without endpoints and we label it by a lower case letter and we say simply *line* $a$. Such a segment is always understood as a line of infinite length.

We could mark any number of points on a line. Points on a straight-line are said *collinear*, e.g. points $A$ and $B$ of figure 3 are collinear. You may add many more points in addition to $A$ and $B$ to line $a$ of figure 3. They are collinear.

**Postulate 3.** Given a segment of a straight-line, a circle can be drawn having the segment as a radius and one endpoint as a center.

This postulate expresses the association of a line segment with a circle so that one endpoint of the segment is the *center* $O$ of the circle and the other endpoint $A$ is on the circle; the segment $OA$ is the *radius* of the circle (fig. 4). It is a practice to label a circle by the letter of its center, such as circle $O$.

By this postulate the circle is the *locus* of points that can be found in the plane at a fixed distance from a fixed point $O$ called the center of the circle.

**Definition:**

If each part of one object coincides exactly with its corresponding part of another object, the two objects are said *congruent* 

(congruence *is to objects as equal is to numbers.*)
Postulate 4. All right angles are congruent.

A right angle is a shape of an object in the plane. We visualize the shape of a right angle by two lines drawn through the same point and a square figure around the point as illustrated in figure 5. Think of a right angle as a shape of an object not as a number of degrees.

Angles in general are not congruent but right angles are always congruent.

Postulate 5. Given a straight line and a point off the line, there exists only one line through that point that never intersects with the given line no matter how far we extend the two lines.

We will encounter a variation to the statement of the fifth postulate but it always expresses the same thing: two lines, such as \( m \) and \( n \) of figure 6, are parallel if they never intersect. Sometimes we say: two lines that intersect at infinity are parallel. Using symbolic notation we write: \( m \parallel n \) (read \( m \) parallel to \( n \)).

Types of lines

A line is always understood to be a straight line unless the type of the line is specified. There are three different types of lines shown in figure 7. The broken line is made of segments of lines attached by their endpoints and each segment differs by its length and its direction from the sides segments. A curve line is continuously changing in direction.

The length of a line extends indefinitely on either side. It is not limited to the drawn portion of the line.

The unit of length

In a practical world a line segment is a distance between two points, which are the endpoints of the segment. The distance between the endpoints of a segment is also called the length, or the measure of the segment.

The unit of length of lines is the meter, it is dived into subunits: the centimeter and the millimeter, among others, are commonly used in this book. The symbols of units of length are written as lower case characters as follows:
m for meter,
cm for centimeter and
mm for millimeter.

The correspondence between the meter and its subunits is as follows:

\[
1 \text{ m} = 100 \text{ cm} = 10^2 \text{ cm} \\
1 \text{ cm} = 10 \text{ mm} \\
1 \text{ m} = 1000 \text{ mm} = 10^3 \text{ mm}
\]

Other units of length inherited from the old British system of units are still used as practical units of length in the United States. The meter is the legal unit of length all over the world including the United States.

II. Geometric construction of segments

In order to perform geometric operations with segments of lines we need to know how to draw lines of the same length, parallel lines, and lines that form a right angle. These operations are geometric construction. To do geometric constructions we need a ruler and a compass. A ruler is a straight edge marked in centimeters and millimeters.

Construction of a segment of a given length

Compass method

Construct a segment \( PQ \) of the same length of a given segment \( AB \) (fig. 8).

- Mark a point \( P \) on the paper.
- Place the pin of the compass at point \( A \) and open the compass so that the tip of the pencil is at point \( B \).
- Without changing the opening of the compass, place the pin at \( P \) and draw an arc.
- Mark a point \( Q \) on the arc and draw the segment \( PQ \).
- Segment \( PQ \) has the same length as \( AB \).

Ruler method

If the measure of segment \( AB \) is specified in units of length, say 5 cm, draw a line with the ruler and mark a point \( P \) on the line. Use the ruler and mark another point \( Q \) on the line 5 cm from \( P \). This method is less rigorous than the compass method but it is practical and is justified by its simplicity. Engineering
graph papers are graduated in centimeters and millimeters and can be used for construction as well.

**Construction of the midpoint of segment \( AB \)**

*Compass method*

- Open the compass a little longer than half of the segment length. Place the pin of the compass at point \( A \) and draw an arc that extends to either sides of \( AB \) (fig. 9a).
- Without changing the opening of the compass place the pin at point \( B \) and draw an arc that intersects with the first arc at either side of \( AB \) (fig. 9a).
- Join the intersection points of the two arcs with a line that intersects with \( AB \) at \( M \) (fig. 9b).
- Point \( M \) is the midpoint of segment \( AB \).

*Ruler method*

Use the ruler to measure the length of the segment. Compute half the length and mark a point on the segment at a distance from one endpoint equal to the half-length you just computed. Label the new point by a letter, say \( M \). Point \( M \) is the midpoint of the segment. This method is less rigorous than the compass method but it is practical and is justified by its simplicity.

**Figure 9.** Construction of the midpoint of a segment.

**Figure 10.** Construction of a line \( m \) at right angle with line \( l \) at point \( P \).