The Wayward Comet

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A Descriptive History of Cometary Orbits, Kepler's Problem and the Cometarium

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Universal-Publishers
Boca Raton

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Universal-Publishers
Boca Raton, Florida • USA
2016

ISBN-10: 1-62734-064-5
ISBN-13: 978-1-62734-064-9

www.universal-publishers.com

Cover design: tatlin.net

Publisher's Cataloging-in-Publication Data

Beech, Martin, 1959-
The wayward comet : a descriptive history of cometary orbits, Kepler's problem and the cometarium / Martin Beech.
pages cm
Includes bibliographical references and index.
ISBN: 978-1-62734-064-9 (pbk.)
1. Comets. 2. Solar system. 3. Outer space. 4. Kepler, Johannes, 1571-1630.
5. Newton, Isaac, 1642-1727. I. Title.
QB721.4 .B44 2015
523.6—dc23

2015915250

This book is humbly dedicated to Professor David W. Hughes,
who has inspired my interest in small solar system objects over many years,
and to all the researchers, planners and engineers who made the European
Space Agency's *Rosetta Mission* to comet 67P/Churyumov-Gerasimenko
such a wonderful success.

"*And a wayward comet was flashing there –*
And he shone upon the sea;
Where a spectre-bark, like a thing of air,
Was moving wearily;
He shot away from sphere to sphere
With a strange and reckless speed;
With a flight of beauty – a flight of fear –
Where none had dared to lead;
And every ray from him that fell,
Went into the soul, like a secret spell." [1]

TABLE OF CONTENTS

INTRODUCTION

Comets have been celebrated and revered throughout all human history, and they are as old as the solar system itself. Indeed, comets, or more specifically the icy nuclei that constitute the heart of the cometary display, are older than the Earth. Literally giant icebergs in space, cometary nuclei were among the firstborn, being the icy planetesimals that grew in the frigid bulk of the nascent solar nebula. Cometary nuclei coalesced in the primordial alchemy that took place 4.56 billion years ago, and they have harassed and hydrated the Earth and planets ever since – indeed, since the proverbial day one they have spread long bedazzling tails across the darkness of the night sky.

Most comets appear entirely at random; they are renegades that work against the clockwork certainty of the celestial sphere. Mercurial, temporary, different and unlike anything else in the heavens, humans could not fail to find comets strange and unsettling – they literally and figuratively rebel against the order and permanency of the stars. Perhaps it was some ancient rumor or mythological-twisting of a past collision that resulted in comets being cast in the role of unwelcome strangers; for certainly, from the earliest of surviving records, the sighting of a new comet has invariably been taken as a cause for concern. We may well wish upon a falling star, but the sudden appearance of a comet is not something to be wished for. Portentous messengers of doom, especially for any ruling aristocracy, Shakespeare reminds us that comets are a special heavenly sign, "when beggars die there are no comets seen; the heavens themselves blaze forth the death of princes". In parallel sympathy, Erasmus, that great medieval humanist, impressed upon society, with both ancient and modern-day resonance, that only by "the good influence of our conduct may we bring salvation to human affairs; or like a fatal comet we my bring destruction in our train". Indeed, comets have long taken the rap for instigating extraordinary earthly proceedings – being cast as the purveyors of famine, the bringers of war, the harbingers of political downfall, and the agents of flood, drought, and plague.

Victorian novelist Thomas Hardy, who was well read in astronomy, further deepens the mystery of comets when he describes their properties as being "erratic, inapprehensible, [and] untraceable" [1]– all characteristics that run counter to the human ideal of a world full of order, understanding and certainty. Indeed, public suspicion (and gullibility) runs deep when it comes to comets. The great Christian apologist C. S. Lewis portrayed a *lurking* comet in his science-fiction work, *Out of A Silent Planet* (first published in 1938): the book's philologist hero Dr. Elwin Ransom seeing, on his way to Malacandra (the planet Mars), against a backdrop of splendid stars and majestic planets, a "comet, tiny and remote". Here the comet image is one of implied malevolence, a cowardly evil ready to stir-up trouble when the time is right. Never was comet-angst and disharmony more rampant than during the 1910 return of Halley's Comet. People fretted over the uncertainty of what might happen as the Earth traversed its extended tail, and the 'snake oil' industry went into overdrive. A comet-born demon was about to be unleashed upon our hapless globe, and only the well-prepared and cautious would survive the ordeal. The comet-sent killer, an invisible and silent stalker, was to be the deadly gas cyanogen (HCCN), a molecule that spectroscopists had previously identified within cometary light. Clearly, many reasoned, therefore, all humanity was to be asphyxiated when the Earth swept through the comet's noxious outgassings – it was literally the sting in the comet's tail. Panic, of a sort, ensued and sugar-coated quinine pills, along with mouth inhalers and gas masks could be purchased to ward off the deadly effects of the comet's emanations. Submarines were made available for hire so that the discerning few might ride-out the encounter under the safety of the ocean waves, for indeed, as one newspaper clipping extolled, "deadly cyanogen gas does not travel through the water". In Arizona "comet proof" rooms were constructed by the Malapai Mining Company – guests, the *Arizona Republican* newspaper for 16 May reported, would be walled-in for 10 days in order to "counteract any poisonous comet gas". For those more inclined to celebrate the end of days, however, other products were made available to ease their would-be passage into the afterlife – for the hard drinker there was Comet Whiskey, especially distilled by Bernheim Brothers in Louisville, Kentucky; the more bohemian of taste could imbibe a Halley Highball or a Cyanogen Flip. If drinking was not a personal preference, then the more temperate in nature could while away their final hours to the rolling refrains of Ed Mahoney's especially composed *Comet Rag*. At other times in history the hapless comet observer could have consoled the desperation of existence by taking a sip of prized comet wine [2] - but history tells us that 1910 was not to be a vintage year. The grapes did not wither upon the vine as Earth passed through the tail of Halley's Comet on 19 May 1910, nor, for that matter, did they especially flourish, but no one died of cyanogen poisoning. Deaths, however, were reported, with a desperate few choosing to take their own lives before cometary devastation descended upon the Earth. Eighty-seven years later the same sad story played itself-out when comet Hale-Bopp came in from the frozen depths of the outer solar system. Lost to the resources of sensibility and apparent reason, 39 members of the Heaven's Gate Cult tragically committed mass suicide on 26 March 1997, convinced that they would be teleported to salvation aboard an alien spaceship that was following the comet towards the Sun [3].

In contrast to the news that was to prevail in 1910 and the Heaven's Gate tragedy of 1997, renowned author H. G. Wells chose to run against tradition in his short novel *In The Days of the Comet* (published in 1906). For Wells the nameless comet was a catalyst, with the storyline seeing the Earth pass through its gaseous tail and thereby ushering-in a wonderful atmospheric transformation. Echoing ideas popularized by Isaac Newton in the early 18th Century, Wells had the comet invigorate Earth's atmosphere: "the nitrogen of the air [was changed into] a respirable gas, differing indeed from oxygen, but helping and sustaining its action, a bath of strength and healing for nerve and brain". In short, because of the comet encounter Earth's atmosphere becomes a kind of *happy-gas*, resulting in the end of war, hatred and strife. The chemistry and transformation, of course, is nonsense, but it enables the story to develop. Indeed, *In The Days of the Comet* is one of Wells's more gentle works [4], evoking as it does, the development of a Utopian society that actually appears to work – a conclusion at odds with his more famous earlier work *The Time Machine* (published in 1895).

Writing several decades before Wells's *The Days of the Comet,* Jules Verne explored the consequences of an Earth grazing encounter (which supposedly took place on "January 1, 188x") in his 1877 novel, *Off on a Comet.* The comet, given the name Gallia, causes a fragment of the Earth to be launched into space, and the story follows the journey undertaken by the hapless survivors. The Earth-fragment undergoes a miraculous and astronomically revealing tour of the solar system, and exactly one year after being launched it returns to the Earth's orbit – at which point the survivors return to *terra firma* by hot air balloon. As with most of Verne's fantasy stories many facts and numbers are presented, but, too admittedly over nit-pick, it is clear that he woefully misunderstood cometary orbits and Kepler's laws. The intriguing aspect of the comet inspired novels by Wells and Verne, however, is that when they were written their somewhat bizarre story lines would have been deemed entirely plausible, or at least not impossible.

Comets have long been the celestial mirrors to reflect human hopes and fears. They have brought-out individual weaknesses, and they have inspired intellectual triumphs. The story of discovery, however, is far from complete and much yet remains to be unraveled with respect to the physical origins and evolution of the wayward slips of light called comets. And, while many volumes have been written about the history of comets, it is not my intention to overly dwell upon that long-explored material here. Yes, comets are special and they certainly have a prominent place in human history – indeed, their unexpected appearance, on occasion, has literally changed the course of history – but here it is the taming of the comet that I wish to study. What, indeed, are the wayward paths followed by comets? More than straightforward geometry, however, it is also the history and development of one specific machine, the cometarium, which will be explored in this text. Indeed, the cometarium is the tamed comet; it is the comet placed in clockwork with its path and appearance constrained to the rhythmical turn of a mechanical crank. As an artifact of that great, if not innate, human desire to build models, both physical and mental, the cometarium is a tactile expression of a comet's curving path – we can literally hold the *dynamics* of a comet within our hands - rotate it before our eyes and change our perspective view like some omnipotent observer. Indeed, the cometarium brings the unimaginable scale of a comet's orbit down to Earth, making it human-sized and perceptible as a single entity. The cometarium turns the complex physics that underlies a comet's motion in space into the indomitable and certain roll of meshed gearing – it literally cuts to the chase. For all this, however, the tameness portrayed by the cometarium, as we shall see, turns out to be a chimera; the comet is a beast that moves beyond the certainty encoded within finely interlocking gears. But all is not lost, and nor were the skills and labor of the instrument-maker deployed in vain; the cometarium, as part of the great panoply of planetary models, is an enduring art form, and a work reflective of great human intellect.

While the original cometaria were constructed using a non-standard set of twin elliptical pulleys, the more contemporary models are made of card-stock, thin wire, and on occasion glass and wood. These latter day cometaria capture, in freeze-frame format, the entire cometary path – the time and position variation of the comet being traced out upon a gently curving arc. Less elegant, perhaps, than their engineered forebears, modern card cometaria are no less useful in showing, even to the simplest tyro, how comets move through the inner solar system. The craftsmanship may well have gone, but the message endures. And, indeed, it is through such simple card models that the general public and the non-specialist have come to *know* the ways of the comet. Largely replaced now by animation sequences displayed on a computer screen, the card cometarium still has contemporary pedagogical relevance, and building, by ones own hand, such a display provides, even in our computer dominated age, a personal, tactile and tangible link to the world of the comet.

At its core the cometarium is a demonstration device – built to illustrate the highly elliptical nature of cometary orbits, and to mimic the phenomenon of changing orbital speeds. Indeed, the cometarium is the mechanical embodiment of the first two laws of planetary motion as outlined by Johannes Kepler in his great thesis *Astronomy Nova* published in 1609. The cometarium, as traditionally designed, however, fails in one of its prescribed tasks. The failure is only slight, indeed, not readily noticeable to the eye, and the failure in no manner distracts from the cometarium's intended demonstration function. The sticking point, for those that wish to quibble, is Kepler's second law. While the cometarium *illustrates* this second law, revealing the characteristic speeding up of a comet's motion as it rounds the Sun, it does not actually solve-for the full requirements of Kepler's second law. The cometarium is not an analog computer. For all this, however, specialist mechanicalcousins to the cometarium were constructed during the early to mid 20th Century in order to solve the equations associated with Kepler's Problem. As we shall see later on, while Kepler's three laws of planetary motion tell us about the orbital shape and characteristic motion of an object (a planet, comet, asteroid, or spacecraft) moving under the influence of a central force, they do not actually tell us where the object will be in its orbit at some specified time. It is this latter question that constitutes Kepler's Problem. In the modern era the equations behind Kepler's Problem are easily and rapidly solved for on a PC, but before the time of electronic computation much pencil-work was required to extract a solution. Necessity, as ever, being the mother of invention, resulted in the design and construction of several remarkable mechanical devices to help speed-up the astronomers' number-crunching. These analog devices, which have no specific group name, were very much scientific instruments; built with the aim of extracting an actual number from a set of variable input parameters. Ingenious, complex, delightful and now entirely obsolete, these devices, just like the original cometarium, had a relatively short ascendancy, but it is their remarkable story that will be revealed in the following chapters.

A few final introductory comments about units and notation are now probably called for. Astronomers notoriously mix and match their units in almost any fashion; in general, however, the *SI* system will be used through this text, with astronomical distances being given in astronomical units (AU), light years (ly) or parsecs (pc). The astronomical unit corresponds to the Earth's orbital semi-major axis and is equivalent to some 149.6 million kilometers. The light year and the parsec build upon the AU scale, with 1 ly = 63,240 AU and 1 pc = 3.262 ly. Comet designations have a somewhat complex history and format, and not every text applies them consistently. Most comets don't have a specific common name, but all comets have a designation number that is now provided for by the Minor Planet Center (MPC) at the Harvard-Smithsonian Center for Astrophysics in Cambridge, Massachusetts. The current designation system is based upon the year of discovery and a letter plus number pair indicating the month and order of discovery. A comet with an orbital period less than 200 years which has been observed to pass perihelion at least twice is given a P/ designation – so Halley's Comet, for example, is known as comet 1P/ Halley since it was the first known periodic comet, and its orbital period was determined by Edmund Halley to be about 75 years. Halley's Comet is also cataloged according to each of its recorded returns, and is additionally designated, for example, as 1P/1982 U1 (the U1 indicating that it was the first comet to be detected in the first half of the month of October). A non-periodic comet or a comet observed just once is given a C/ designation – so, for example, the recently observed comet Hale-Bopp (discovered by Alan Hale and Thomas Bopp) is designated C/1995 O1 (with the O1 indicating that it was first detected in the second half of the month of July). A lost and/or destroyed comet is given a D/

designation: comet D/1894 F1 (Denning), for example, was discovered by the famed amateur astronomer William Frederick Denning in March of 1894, and although the observational data at that time indicated an elliptical orbit and an orbital period of about 7.5 years, the comet has not been seen since Denning first swept it up. And, finally, in terms of applying names to comets, the current rules set by the MPC dictate that up to three independent discoverer names can be applied to any one comet.

CHAPTER 1
THE COMET CONSTRAINED

The education of Euphrosyne

Obscure in the modern-era, the heyday of the cometarium was set in the first half of the 19th Century. It is a device that was built upon the intellectual certainty of Newtonianism and the observational triumphs of 18th Century astronomy. Before exploring the details of all these multifaceted connections, however, let us first see how famed instrument maker and jubilant author Benjamin Martin (1714 - 1782) introduced the cometarium in his imagined dialogue between a young natural philosopher, Cleonicus, and his eager and able pupil, Euphrosyne (figure 1.1). The year of the work is 1772, and Halley's Comet had but 13 years earlier appeared in the night sky to vindicate Edmund Halley's 1705 prediction and thereby sealing the hegemony of Newton's theory of gravitational attraction as applied to celestial bodies. The dialogue proceeds thus:

Figure 1.1: *The Young Gentleman and Lady's Philosophy* – Dialogue XVL by Benjamin Martin (1772)

Euphrosyne: Since you gave me the lecture on comets, you have filled my head with such odd kinds of ideas, and I scarcely known whether I hope or fear most to see a comet; but dear Cleonicus, since that is shortly to be the case, and a comet we must behold, if your astronomical prediction is to be regarded, I think I may as well take courage, and resolve to attend the important event undauntedly.

Cleonicus: Fortitude, my Euphrosyne, is an excellent virtue; and hence I must admonish you to speak with more reverence of astronomical predictions.

Euphrosyne: If I remember right, you once told me, that you could make the manner of the comet's motion intelligible by a proper instrument, as well as those of the planets.

Cleonicus: I did so; the instrument I mean is called the cometarium, and which I shall now spend one quarter of an hour in explaining to you – Here is the machine.

Euphrosyne: And a beautiful one it is; I can almost tell the use of it by its very appearance.

Cleonicus: Observer, when I turn the winch, the brazen comet moves, and with a very unequal pace in its elliptical orbit, about the focal Sun.

This wonderful dialogue, in just a few short refrains, sets the scene for the appearance and application of the cometarium, invented, in fact, some forty-years before Martin was writing, and it also brings out the new order that had been imposed – or more correctly revealed – in relation to cometary orbits. Indeed, the final stanza of the discussion presented points towards serious astronomy and the visual illustration of the first two of Kepler's laws of planetary motion. In contrast, however, the first stanza reveals an initial sense of doubt on behalf of Euphrosyne. The fact that the prediction is to be regarded as correct, as emphasized by Cleonicus rests upon Newton's laws and the (mostly) correct predictions by Edmund Halley in 1705 (to be described below).

But what was this device that Cleonicus showed to Euphrosyne? The answer to this lies partly within an earlier work by Martin. Businessman that he was, Martin knew that the predicted return of Halley's Comet in 1758 was bound to cause a ripple of public interest in matters astronomical. To this end he published in 1757 a small pamphlet entitled *The Theory of Comets Illustrated*, and to go with this work he re-invented and re-named a device previously called the equal-area machine; a demonstration device made known through the popular public lectures of Scottish astronomer James Ferguson. It was Martin, in 1757, who first coined the name cometarium, and for the tidy sum of just 5-Guineas such a device could be purchased to further enhance ones viewing of the spectacle soon to be portrayed in the heavens above.

From the very outset the appearance of the cometarium is somewhat odd and perhaps even intimidating (figure 1.2). Its working face displays a number of graduated dials and pointers, and set within a large ring is an elliptical track-way. Certainly it has the appearance of a serious demonstration device - there are no frills or extraneous details. But what exactly is the device trying to tell the user?

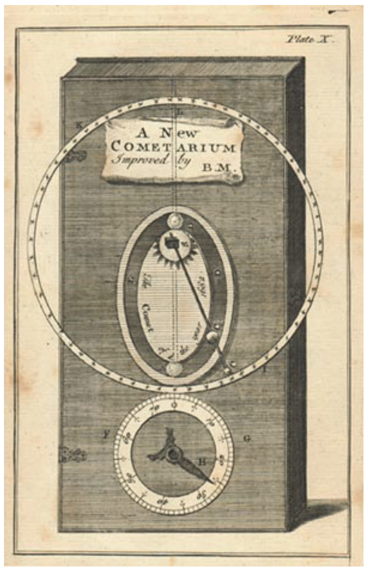

Figure 1.2: The cometarium as improved by Benjamin Martin. The text around the elliptical track reads: "the come of the year 1682".

The cometarium is not exactly like the planetarium and orrery, whose function is essentially evident at first glance; they are devises to show the relative motion of the planets around a central Sun (see Chapter 2). This being said, some features of the cometarium, as Euphrosyne so eloquently observed, are readily understandable. Having already been told that comets move along elliptical orbits about the Sun, it is clear that the central elliptical track must represent the path of the comet. Indeed, inscribed around the edge of the inner elliptical plate are the words "the Comet of the year 1682". This track, therefore, represents the orbit of Halley's Comet, and indeed, the year corresponds to the very time at which Halley observed the comet and began to wonder about its origins and past history (see Appendix 1).

Given that the elliptical track corresponds to the orbit of the comet the rotation point of the comet-driving radial arm must correspond to the location of the Sun - and, indeed this rotation point is distinguished by a spiked coronal motif. From this offset position of the Sun, it is immediately seen that there are two points in any comets orbit where it is at its closest and most distant locations. These are the perihelion and aphelion points respectively, and it takes the comet exactly half of its orbital period to move from one to the other. It is the rotation of the radial arm extending form the Sun focus point that drives the comet ball around the elliptical track, and the large circular ring, centered on the Sun, corresponds to the great circle of the comet path projected onto the celestial sphere. The lower circular dial of the cometarium indicates the time elapsed since perihelion passage, and its scale is divided into 75 divisions, corresponding to an orbital period of 75 years.

It is now well known that the period of Halley's Comet is not exactly 75 years, and that from one perihelion passage to the next its orbital eccentricity and orientation can change. The cometarium is not intended to represent, therefore, any specific orbit as it might have been observed at some specific return of Halley's Comet, nor does it act as a predictive instrument to tell the user where the comet is going to be in the sky. Indeed, although Martin labeled his cometarium as corresponding to that of the comet for 1682, this designation is entirely irrelevant, and in reality any orbital period can be modeled. The only relevant point is that one 360-degree rotation of the time dial corresponds to the orbital period of the comet. The only way that different orbital eccentricities can be accommodated is by literally changing the eccentricity of the track in which the comet ball is constrained to move. This being said, Halley's Comet was the very first verified periodic comet, and in 1755 it was still the only comet known to have returned more than once to the inner solar system.

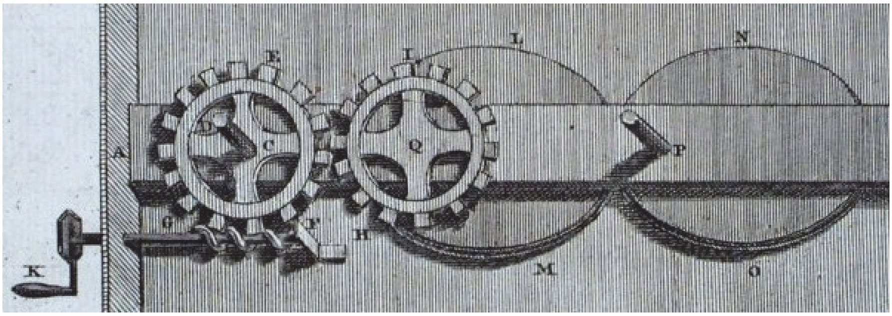

Figure 1.3: The interior wheel-work to Benjamin Martin's cometarium.

To make the cometarium work the demonstrator turns the small crank K (as seen in the lower left hand corner of figure 1.3). Connected to a continuous worm gear, the crank directly engages with the first circular gear c. The center of gear c is connected to the lower time dial located on the front of the cometarium. The crank K, therefore, is the time input of the machine; the more turns being given to crank K, so the greater the time interval being recreated along the comet's orbital path. Gear C is directly meshed to a second circular gear Q whose center is located at one of the focal points of the elliptical track. The spindle at Q is further fixed to a rigid elliptical former LM. If we now imagine that the demonstrator turns the crank K at a constant rate, the gearing - so far described - is such that the elliptical former will be driven about its focal point at a constant rate. This constant rate of rotation is transformed into a non-constant rate of rotation by letting the elliptical former LM drive a second elliptical former NO about its focal point P. This drive is achieved via a figure of eight catgut string constrained to move along a v-grove cut into the edges of the elliptical formers. A spindle attached at P to the elliptical former NO will now rotate at a non-constant rate. By attaching a drive arm to the spindle at P on the front face of the cometarium, the comet ball will be driven around its track with varying speed. Spindle P represents the Sun-occupied focal point of the comet's orbit, and the drive rate of the comet ball will be at its fastest when near to the Sun and at its slowest when far away from the Sun. Further details of the motion of the comet ball will be presented later in Chapter 2 and in Appendix II. In the mean time, we now regain contact with the dialogue of Cleonicus and begin to understand his final quoted words which tell us that the "brazen comet moves, and with a very unequal pace in its elliptical orbit about the focal Sun". The cometarium being described by Cleonicus is clearly, given the focal point location of the Sun, a product of the Copernican hypothesis and it is the development of this idea that we shall briefly consider next.

The workings of the solar system: a brief history

Although the night sky is animated by the stately movement of the stars and attendant constellations, its constancy and very predictability is not absolute. To the human eye there are seven objects that behave differently to the stars - these are the Sun, the Moon and the planets Mercury, Venus, Mars, Jupiter and Saturn. These objects, vastly different in brightness, and vastly different in their daily motions, move across the backdrop of the stars - they do not move with them. The Sun, Moon and planets are obvious oddities when compared to the pinpoints of light that pockmark the celestial vault and move in lockstep unison, in mythical shapes and asterisms, rising in the east and setting in the west, but never changing their positions one relative to another. To the planets can be added the wayward motion of comets and the transitory flashes of meteors, although the fact that these latter objects even fell within the realm of astronomy is something of relatively recent genesis (see Appendix I).

To the ancients, well versed in common sense, it seemed obvious that the Sun existed to illuminate the day and the Moon to periodically illuminate the night - but what of the planets? What was their purpose, and why did they move differently and with a brightness that varied from one month to the next? By considering the stars[1] human insight moved in two, polar opposite, directions and laid the foundations for what later, much later in fact, became the stout and resolute body of science and its mercurial shape-shifting off-spring astrology. Both areas of study are still with us today, but science, the practice of measured and rationale thought combined with prediction and experimentation, as applied to natural phenomena, is now in the ascendancy. For all their differences, however, each discipline, in its own way, has had much to say about the appearance and properties of comets.

The Greek philosophers championed the early ideas, and Claudius Ptolemy, in the first century AD, gave us a summary of all that had been deduced. Ptolemy's great work *Syntaxis Mathematica,* better known through its Arabic translations as the *Almagest*, was a *tour de force* - a superb encyclopedic work, full of brilliant, if not controversial ideas. For the ancients, the Earth stood fixed and un-moved – a spherical body at the center of the universe. Around the earth, in a series of concentric shells, were the assembled planetary realms, and about them all was the sphere of the stars. Ptolemy provided a set of detailed mathematical instructions to determine the motion of the planets, moon and Sun. Using an eccentric offset, epicyclic construction, Ptolemy was able to provide an excellent description of the motion of all the planets as they moved across the celestial sphere. He achieved this superb description, however, by introducing the idea of the equant point (figure 1.4). The equant point, while vital for making the Ptolemaic system work, was roundly criticized by subsequent commentators - it was a step too far since it gave importance to an otherwise empty point in space and it introduced the requirement of another point (the center of the epicycle) moving with a non-constant velocity into celestial calculations. The latter attribute went firmly against Plato's original doctrine that all celestial motion should precede with a constant velocity and within a perfect circle. The circular motion component was contained in Ptolemy's model, but to describe such observed characteristics as retrograde motion, and the periodic variations in the accumulated motion of a planet across the sky, in equal intervals of time, the equant was vital.

While the Arabic and medieval astronomers did all they could to reduce the number of epicycles and to remove the equant from planetary theory: "epicycles correspond to nothing in nature" decried Henry of Langenstein in his 1373 *Contra Astrologos*. Ptolemy's model worked well, indeed, it worked exceptionally well, but it offered no harmony of thought and no consistency of concept. At issue, ultimately, was the question of reality; how are the planets really distributed in space, and where exactly is the Earth located with respect to the other celestial objects. A purely mathematical description of planetary motion, such as that offered by Ptolemy, was all well and good, but

[1] We note that the underlying etymology of 'consider' is the Latin *considerare*, meaning 'to observe and reflect upon the stars'.

did the mathematical model actually describe the reality of the heavens and God's creation.

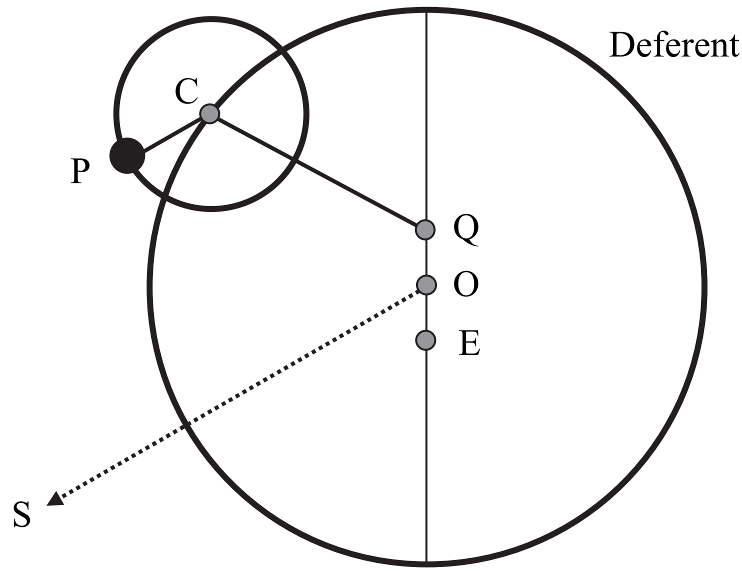

Figure 1.4: Schematic diagram of Ptolemy's planetary theory. The center of the deferent is located at O, while the Earth and the equant are located at E and Q. The center of the epicycle C moves around the deferent such that it sweeps out equal angles in equal intervals of time about the equant point. The planet P is positioned on the epicycle according to the requirement that CP is parallel to OS, where S indicates the location of the (fictitious) mean Sun which moves with a constant speed about the center O, completing one full rotation in the time interval of one year.

Ptolemy's *Syntaxis* was THE astronomy book for nearly one and a half thousand years - an incredibly run by any standards. His work provided a practical means for determining the positions of the Sun, moon and planets at any time, past, present and future, but as the centuries ticked by it was increasingly viewed as an esthetically unpleasing system. It was for these latter esthetic reasons, rather than because of any predictive deficiencies, that Nicolaus Copernicus set out to redefine and reshape the planetary realm in the mid-16th Century. He worked upon his ideas for nearly half of his life, but encouraged by his young disciple Georg Rheticus, eventually published his magnum opus, *De Revolutionibus Orbium Coelestium*, in 1543.

Copernicus's work was altogether something different, not because it was actually new, other philosophers, at various times, had suggested similar such ideas, but because Copernicus actually worked out the mathematical details.

In his design, however, Copernicus looked backward to the postulates of Plato which required that planets must move with uniform speed along circular paths (or orbits as we now call them). This thinking was in some sense regressive, but by placing the Sun at the center of the universe (as Copernicus knew it) and making Earth the third planet out, then a reasonably good description of observed planetary motion could be obtained. The accuracy of the initial Copernican model was inferior to that of the Ptolemaic model, but as he had intended all along the model developed by Copernicus was simpler in concept and it provided a uniform description of the heavens. Each planet in the Copernican system had its own specific orbit and the spacing between the planets could be determined directly from the observations.

It was the straightforward construction of the Copernican model that caught the imagination of other philosophers, but it took the mathematical genius of Johannes Kepler to make the model work. Building upon the highly accurate positional data obtained by Tycho Brahe, Kepler applied what in modern terminology would be called a reverse engineering study. Specifically, Kepler used the observational data to determine the shape of the orbit of Mars rather than assume, as Copernicus had, that it must be circular. Finding that the orbit of mars was elliptical, not circular, then automatically required that the speed with which a planet moved along its orbit must vary, being most rapid when close to the Sun and slower when further away. Not only this, the observational data indicated that the Sun must be located at one of the two focal point positions of a planet's elliptical orbit (figure 1.5). Kepler explained the first two of his laws in *Astronomia Nova*, published in 1609, and later introduced a third law, in a somewhat obscure form, in his *Harmonice Mundie*, published in 1618. Kepler's collected laws of planetary motion as we now known them are:

K1: The orbit of every planet is an ellipse with the Sun located at one of its focal points

K2: A line drawn between the Sun and a planet sweeps out equal areas in equal intervals of time

K3: The square of the orbital period is proportional to the semi-major axis cubed

It is K1 that describes the shape of planetary orbits and identifies the location of the Sun with respect to the orbit (see figure 1.5), while it is K2 that describes how the planet moves in its orbit and explains why the speed of the planet varies between perihelion and aphelion. When first announced in his *Harmonice Mundie* K3 was an absolute mystery - there was, at that time, no explanation as to why such a relationship between the orbital period and orbital size should exist. While Kepler tried to explain his laws in terms of magnetic planets interacting with a magnetic monopole Sun, it was Isaac Newton who eventually provided the first correct physical reasoning behind all three of the planetary laws. Indeed, Newton showed that any two objects interacting under a centrally acting force that varied as the inverse square of distance must automatically obey Kepler's laws (figure 1.6). The force that Newton introduced, of course, was that of gravity, and this combined with the laws pertaining to the conservation of energy and angular momentum provided a full description of celestial dynamics. These remarkable results were first articulated in Newton's masterpiece, *Philosophiæ Naturalis Principia Mathematica*, published by the Royal Society of London, with financing by Edmund Halley, in 1687. Not only was Halley the paymaster and editor of Newton's remarkable text, he was also the inspired astronomer who wrote an ebullient dedication to the tome's esteemed author. Indeed, Halley eulogized that, "now we know the sharply veering ways of comets, once a source of fear and dread, no longer do we quail beneath appearances of bearded stars". In the same way that the pen is mightier than the sword, so Newton's mathematical analysis and physical insight were mightier than two thousand years, and more, of superstition and fear (see Appendix I).

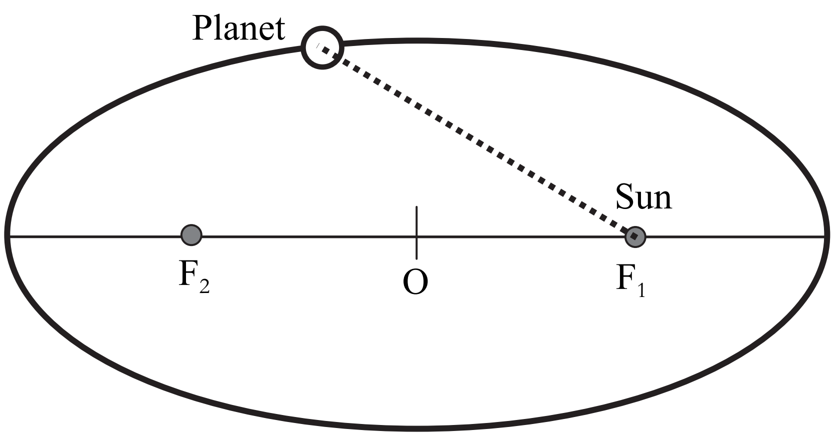

Figure 1.5: The key characteristics of Kepler's 1st law are that the orbital path is elliptical in shape and that the Sun is located at one of the two focal points (F_1 and F_2) of the ellipse. The two focal points are situated at equal distances away from the center (O) of the ellipse. A line drawn between the planet (or comet) and the non-Sun, or empty focus, has dynamical characteristics similar to that of Ptolemy's equant point.

Coda: Newton's proof of K2

With reference to figure 1.6, imagine an object at point B subject to the gravitational pull of an object located a point S. The arc ABCDEF indicates the successive positions of the object at equal time intervals *t*. In the first time interval, the objects moves from A to B and sweeps out the area SAB – at this stage the motion is rectilinear and the object moves along the straight line path AB. Once at point B, however, the gravitational force begins to pull the object away from its straight line path with the result that it moves along the path BC rather than Bc. The proof of Kepler's 2nd law now proceeds by demonstration that in the second time interval the area swept out by the object SBC, as it moves from B to C, is exactly equal to area SAB. Firstly, Newton noted that the area of triangles SAB and SBc are equal – this follows since they have the same altitude (the perpendicular line dropped from S to the line extending through ABc) and they have the same base lengths: AB = Bc. The next step is to show that the area of the triangle SBc is equal to SBC. This is accomplished by noting that the displacement cC (due to the gravitational force acting at S) runs parallel to the line SB. With this condition in place Newton had his proof of equal areas since the two triangles of interest have the same base length SB and identical altitudes (the line dropped from c – and C – to the line extending along SB). Having shown that triangles SAB and SBC have equal areas, the final part of the proof is just a generalization – the same result, as just proven, must apply to the motion of the object from point C, with the triangles SBC and SCD also having equal areas.

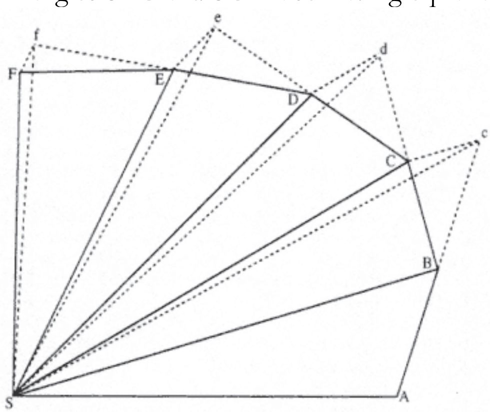

Figure 1.6: Newton's diagram explaining the motion of an object under a central force and his proof of Kepler's second law. Newton's proof of K2 proceeds geometrically and requires a demonstration that the area of triangle SAB is the same as triangle SBC, which is the same as SCD and so on.

Newton's proof of Kepler's second law is a remarkable geometric construction – he has employed nothing more than straight lines and triangles, and the result is independent of the actual value of the time step *t*. Likewise the proof is independent of the magnitude of the centrally acting force at S [1].

Comet C/1680 V1 - the game changer

It took over 70 years to come about, but eventually, on 14 November 1680 German astronomer Gottfried Kirch became the first person to discover a comet with the aid of a telescope. Working in the early morning hours, and using a 2-foot focal length refractor Kirch was observing the Moon and Mars when by chance, in the constellation of Leo, he sighted "a sort of nebulous spot, of an uncommon appearance….. a nebulous star, resembling that in the girdle of Andromeda[2]". Kirch followed his nebulous star over ensuing nights, and on November 21st using a 10-foot focal length telescope confirmed the appearance of a small but distinct tail. The comet was increasing in brightness and heading towards the Sun. By the close of November the new comet was visible to the naked-eye and its tail was reckoned to be over 15° long.

As December proceeded the comet grew ever brighter but by mid-month it was lost to view within the Sun's glare. Perihelion occurred on December 18th. Rapidly rounding the Sun the comet re-emerged to view sporting an extraordinary long, near 90° tail on December 20th. Many years later Augustan missionary Casimo Diaz recalled his sighting of the comet from Manila in the Philippines: "the frightful comet [was] so large it extended, like a wide belt, from one horizon to the other… causing in the darkness of the night nearly as much light as the Moon in her quadrature".

Having caught the eye of the world's populace the Great Comet of 1680 now required an explanation from the astronomers. The first question that needed to be settled was whether one or two comets had actually been seen. Some observer's, Isaac Newton among them, initially argued that two comets has been seen: the first being Kirch's comet heading inwards towards the Sun, with a second comet, purely by chance emerging from behind the Sun on December 20th after Kirch's had disappeared from view. Other observer's, Britain's Astronomer Royal John Flamsteed foremost among them, argued that just one comet had been seen, and that remarkably it has been repulsed by the Sun and set upon a parabolic path back into the outer realms of the solar system. Much debate followed, but it appears that it was not until at least mid-1684 that Newton came around to the viewpoint that comets might move along elliptical orbits and that the comet of 1680 had, "fetched a compass about the Sun". Applying his formidable intellect to the problem, Newton developed the mathematical techniques needed to deduce, from a set of three evenly spaced observations, the parameters that describe the shape of a comet's orbit (to be discussed below). The fruit of Newton's labors were eventually revealed in Book III of his 1687 *Principia* (figure 1.7), where he demonstrated that the Great Comet of 1680 had traveled along a parabolic path with, importantly, the Sun located at the focal point. Newton had not only shown how to deduce the path of a comet from the observations,

[2] The nebula in Andromeda corresponds to the Andromeda Galaxy (M31).