ΜΗΔΕΙΣ ΑΓΕΩΜΕΤΡΗΤΟΣ ΕΙΣΙΤΩ

(Do not enter if you do not know geometry)
This book is addressed to students and professionals and it is aimed to cover as much as possible the wider region of topographic mapping as it has been evolved into a modern field called *geospatial information science and technology*. More emphasis is given to the use of scientific methods and tools that are materialised in algorithms and software and produce practical results. For this reason beyond the written material there is also a large number of educational and professional software\(^1\) programs written by the author to comprehend the individual methodologies which are developed. Target of this book is to provide the people who work in fields of applications of topographic mapping (environment, geology, geography, cartography, engineering, geotechnical, agriculture, forestry, etc.) a source of knowledge for the wider region so that to help them in facing relevant problems as well as in preparing contracts and specifications for such type of work assigned to professionals and evaluating such contracting results. This book is also aimed to be a reference of theory and practice for the professionals in Topographic Mapping.

The material is carried out by 11 chapters and two appendices as follows:

1. Introduction Background
2. Reference systems and Projections
3. Topographic instruments and Geometry of coordinates
4. Conventional construction of a topographic map
5. Design and reproduction of a thematic map
6. Digital Topographic mapping – GIS
7. Digital Terrain Models (DTM / DEM)
8. Topographic mapping with GPS
9. Topographic mapping with methods of Photogrammetry
10. Topographic mapping with methods of Remote Sensing
11. Topographic mapping with new technologies LIDAR, IFSAR

A. The method of Least Squares adjustment

B. Description of educational software accompanying the text.

This book applies a didactics method where with a relatively small effort someone can digest a quite large volume of simple or complicated material of knowledge at a desirable scientific depth within a relative short time interval. This book is roughly 750 pages and a conventional book with about same material could probably need more than 5000 pages. This didactics method is based on a series of educational software developed by the author (some of these modules can also be used for professional applications) and cover most crucial points on topographic mapping. The reader of this book has the opportunity to follow the scientific analysis of these crucial points, as well as, the process of converting then into algorithms similar to those used by professional software packages. At the same time it is available to the reader corresponding educational software (written by the author) which can immediately be used to verify the correctness of scientific analysis and in this way it is developed a self-confidence for the acquisition of

\(^1\) See last page of this volume for instructions to access the educational software files online mentioned in the book
knowledge, as well as, a powerful motive for further deepening on the scientific subject. This didactics approach is applied by the author since 1980 in his academic and professional career in USA and in Greece with excellent results. The objective that educated people must be “smarter than the machine” and not to treat the machine as a “black box” being “button pushers” has been achieved with relative success by adopting this didactics technique.

Desire of the author is the knowledge acquired by reading this book to be used correctly. For this aim is proposed ten foundation bases listed bellow which may contribute along this direction:

1. **Education** is the effort to develop a healthy thought to the virtuous person.
2. **Training** is the effort to develop a healthy thought to the virtuous person in a specific subject, for example, Topographic Mapping.
3. **Thought** is a composite of three states of human mind before it develops any action. These states are: logic, desire and anger. Thought determines all actions of a person and precedes to these actions. Any state of mind (joy, happiness, sorrow, imagination, etc.) is described by these three components.
4. **Healthy thought** is developed when there is an effort so that the logic state of a person’s mind balances the two other states which are the desire and the anger. If logic state does not balance desire and anger then there is a bias or human error. Balancing takes place within tolerance limits as defined by midway of virtue (within the boundaries of wrong / right).
5. **Healthy action** is the effort so that such action is shaped by a healthy thought and is virtuous.
6. **Virtue** is the effort so that the action of a person follows a midway path, which is found in midway between two extreme positions or badness. For example, thrifty is a virtue and is found in midway between stinginess and overspending. Virtue can be considered as "the effort to maximise the use of mind energy for constructive aims and minimise its use for destructive aims". Some times though, destructive actions are inevitable as for self defence.
7. **Virtuous person** is the one who tries to be virtuous, that is to say, the effort to maintain the midway of virtue. This definition is within human dimensions and allows to anybody at anytime (never is late) to be virtuous.
8. **Virtuous action** is the effort so that corresponding action follows the midway of virtue.
9. **Justice** is the top virtue and includes all virtues.
10. **Democracy** is the process to define the midway of virtue. Consequently, for defining the midway of virtue it is required a wider consensus of educated people with minimum bias or error.

Notice: The Aristotelian midway of virtue has a universal validity, for example, taking into consideration the orbit of the earth around the sun, one may observe that the earth will never follow exactly the same path and there is a midway where orbits of the earth must occur in order to have equilibrium. If the earth gets off such bounds towards the inside (negative error), then the earth may collide with the sun, if the earth gets off such bounds towards the outside (positive error), then the earth may get lost in space. This idea provides a substantial help to define precisely the boundaries of wrong and right.

---

3 Plato “The republic”
4 Aristotle “The Nikomachean Ethics”
To my wife Nini

and to my children

Nafsika, Nikolaos and Despina
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About the Author
CHAPTER ONE

INTRODUCTION - HISTORY
Chapter 1

Introduction History

1.1 General

Topographic mapping of earth’s surface began when exploitation of such areas occurred. Population increase created the need for people to share exploitable areas for urban development, agriculture, forestry, rangelands, etc. In the beginning boundaries were placed around wider areas by isolated groups of people and later such boundaries were placed by families. Then those areas were subdivided to provide land to descendants of such families and so on which means that land parcels were becoming smaller after subdivision. Then restrictions were imposed for subdivisions and exploitation means to arrive in today’s situation where land planning and sustainable management determines land uses while cadastre makes a full description of shape size and attributes of public property. Topographic mapping is expanded beyond needs for exploitation of areas of earth’s surface and it constitutes the basis for management, analysis and development of geographical space. However, within the framework of development interventions on earth’s surface for construction of works such as: roads, public works, cities, communication networks, networks of energy, water and sewer networks, irrigation networks, etc., use topographic mapping as a basis for planning, design and execution of such works. Furthermore, analysis and management of geographical space especially for environmental management, geology, agriculture and forestry, also use as a basis topographic mapping.
1.2 Definitions

Today a parcel is considered as entity or object and it is defined by its geometrical structure such as: size, orientation, dimensions, etc., as well as, its attributes such as: its use, ownership, tax data, etc. Topographic mapping as a science mainly deals with collection of geometric data and monitoring of attributes of similar to a land parcel entities.

Topographic mapping is defined as: *Art, science and technology to locate points near the earth’s surface, to derive geometric structures from these points and to monitor a set of static and dynamic attributes associated with these structures.* This definition includes entities or objects which have a geometrical structure which is composed of points. It must be emphasized that a point is a generic element which is able to form basic and complex geometrical structures such as lines, polygons, areas, parcels, etc. A line, for example, is composed of a set of points ordered along the line and they form the line, the same happens to a surface, it is also composed of a set of points which form the surface. Topographic mapping virtually helps to define points near the earth’s surface which are necessary to map part of the earth’s surface or the entire surface of the earth.

*Geodesy*, on the other hand *is mainly occupied to define the size and shape of the earth* in order to create a reference surface for topographic mapping.
1.3 History

Topographic mapping, according to known historical evidence, existed in ancient Babylonia and the borders between neighboring properties were delineated with specifically shaped idols made of clay which were used as landmarks and were named “boundaries” and were worshiped and protected like gods. The importance of this protection and worship was to decrease disputes and conflicts that usually result from locomotion of property borders and so far it was not allowed to anybody to touch the boundary. In Figure 1.1 is given such a picture of a boundary monument from ancient Babylonia. This example also shows the importance of the “point” as a basic geometric entity (object) for topographic mapping.

Figure 1.1. Boundary (Kudurru of Melishihu) of ancient Babylonia which constitutes the basic geometric entity (object) in topographic mapping. Grey limestone (1202-1188 B.C.).

The original Kudurru was sculptured on stone and was kept within a temple while a replicated idol made of clay was given to the owner and consequently was precisely placed at property limits. Information sculptured on the boundary was dealing with decree of ownership and specific consequences which may result from any effort to alter such decree which is protected by gods.

Some of those consequences are as follows:

_May all great gods whose names are written in this stone, will drive him in a great sadness. Sadness may hit himself and his decedents who may be accursed through the mouth of all people. The name of this stone is “the establishment of boundary forever”._

Figure 2. Map of the earth from ancient Babylonia about 1000 B.C.

The oldest map was found in Babylonia and is dated around 1000 B.C. and it is shown in Figure 1.2. Notice that this map shows the earth’s shape to be round.
Ancient Greeks had made significant progress in topographic mapping and geodesy with top geodesist being Eratosthenes, named father of geodesy, who measured with high precision the size of the earth. Eratosthenes (274 – 196 B. C.) measured with high precision the earth’s perimeter in a simple way as follows: As shown in Figure 1.4, at a given moment the observer watches the reflected idol of the sun in a well located in the city of Seine in Egypt. This means that at that moment the Zenith angle of the sun $Z_1$ was zero ($Z_1 = 0$).

![Figure 1.3](image1.jpg) Eratosthenes (274 – 196 B. C.) who is considered as being the father of geodesy.

At the same time the sun’s zenith angle was measured in Alexandria also city of Egypt which is located at a distance 5000 stadiums away (4878 stadiums with today’s measurements) from the city of Seine and it was found $Z_2 = 7^\circ 12'$.

![Figure 1.4](image2.jpg) The simple way Eratosthenes computed the earth’s perimeter.

Eratosthenes has also measured the angle between the earth’s axis and the plane of the earth’s orbit around the sun (ecliptic) and found to be $23^\circ 30'$ which is very close to today’s measurements $23^\circ 27'$. Beyond Eratosthenes impressive accuracy of measurements, there are many other ancient Greeks who contributed in various ways to topographic mapping. Some names will be mentioned in brief as follows:

Taking into consideration that the Attica stadium is 164 meters, then, the earth’s perimeter was computer by Eratosthenes and was found to be of 41,000,000 meters. If we take today’s measurements of same distance being 4878 stadiums, then the earth’s perimeter is computed as 40,000,000 meters. Taking an average radius of the earth as 6,367,444.5 meters which is adopted by WGS84 and used by GPS, then the earth’s perimeter is computed to be 40,000,7834 meters. This brings down the error from Eratosthenes observations to 7834 meters.