Some Notes on Game Bounds

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ISBN: 1-58112-021-4

DISSERTATION.COM

1998

UNIVERSITY OF CALIFORNIA
BERKELEY

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A thesis submitted in partial satisfaction of the

requirements for the degree Master of Arts

in Mathematics

by

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1991

I am very thankful to Professor E. Berlekamp for suggesting me this material and for some very fruitful discussions.

I thank also JNICT-Programa CIENCIA for their support.

This work is dedicated to
Laura, Manuel and Raquel.

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Abstract

In this work we establish some game bounds. For each finite birthday, N, we find the smallest positive number and the greatest game born by day N, as well as the smallest and the largest positive infinitesimals. As for each particular birthday we provide the extreme values for those types of games, these results extend those in [1, page 214].
The main references in the theory of combinatorial games are ONAG [1] and WW [2]. We'll use the notation and some fundamental results from WW—mainly from its first six chapters—to establish some bounds to the size of the games.

Chapter 1

Basic Definitions

Given a game $G = \{L \mid R\}$ where L, R are two sets of games we'll write $G = \{G^L \mid G^R\}$, so G^L (G^R) will stand for the typical Left (Right) option of G.

Definition 1 *The sum of two games $G = \{G^L \mid G^R\}$ and $H = \{H^L \mid H^R\}$ is defined by*

$$G + H = \{G^L + H, G + H^L \mid G^R + H, G + H^R\}\,.$$

Definition 2 *The negative of a game G is*

$$-G = \{-G^R \mid -G^L\}\,.$$

We write $G - H$ for $G + (-H)$.

Definition 3 *A game in which the second player to move is the winner is a zero game,* 0.

Definition 4 *The partial order is defined by (see ONAG, page 78, for a more formal approach)*

$G > H$ (G is bigger than H) iff $G - H$ is won by Left , whoever starts.

$G = H$ (G is equal to H) iff $G - H$ is a zero game.

$G < H$ (G is smaller than H) iff $G - H$ is won by Right , whoever starts.