

Fundamental Concepts in Electrical and Computer Engineering with Practical Design Problems

**Reza Adhami
Peter M. Meenen, III
Dennis Hite**

Universal Publishers
Boca Raton, Florida

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Universal Publishers
Boca Raton, Florida • USA
2007

ISBN: 1-58112-971-8 (paperback)
13-ISBN: 978-1-58112-971-7 (paperback)

ISBN: 1-58112-973-4 (ebook)
13-ISBN: 978-1-58112-973-1 (ebook)

www.universal-publishers.com

Preface

Purpose

In many cases, the beginning engineering student is thrown into upper-level engineering courses without an adequate introduction to the basic material. This, at best, causes undue stress on the student as they feel unprepared when faced with unfamiliar material, and at worst, results in students dropping out of the program or changing majors when they discover that their chosen field of engineering is not what they thought it was. The purpose of this text is to introduce the student to a general cross-section of the field of electrical and computer engineering. The text is aimed at incoming freshmen, and as such, assumes that the reader has a limited to nonexistent background in electrical engineering and knowledge of no more than pre-calculus in the field of mathematics. By exposing students to these fields at an introductory level, early in their studies, they will have both a better idea of what to expect in later classes and a good foundation of knowledge upon which to build.

Organization

This text consists of twenty chapters organized into five distinct units. In the first unit, the student is presented with a review of some key mathematical concepts. This overview is designed to fill in any gaps in the student's previous mathematical background and to provide a foundation for their study of the remaining chapters of the text. The first unit consists of three chapters. Each of these chapters focuses on a different important area of mathematics with which the beginning engineering student should be acquainted. The topics, covered in the first three chapters of the text, include, numeric bases, complex numbers, and basic elements of vectors and matrices.

Chapter 1 introduces the student to the concept of numerical bases. They will learn what a base is, how to convert back and forth between a given base and decimal, and how to work with some of the most common numerical bases, including binary, octal, and hexadecimal.

Chapter 2 covers the basics of complex numbers. The student will learn what a complex number is, how to represent these numbers in both the Cartesian and polar coordinate systems, how to convert between these systems, and how to perform basic arithmetic operations using complex numbers.

Chapter 3, the final math review chapter, introduces the student to some basic concepts in linear algebra. The chapter begins with the introduction of vectors and some simple vector arithmetic. The focus is then expanded to include matrices. Basic matrix arithmetic, such as addition, subtraction, and multiplication, is covered. Other basic matrix operations are also introduced. These include the concepts of matrix transpose, minors, cofactors, determinants, and inverses. The chapter concludes with an application of the matrix inverse. The use of the matrix inverse to obtain the solution of systems of linear equations is covered and related to the practical engineering application of circuit analysis.

Once the basics in mathematics have been established, the student may move on to the other four units of the text. The second unit consists of three chapters that are designed to provide the student with a better understanding of basic concepts in the area of electrical engineering.

Chapter 4 focuses on providing the student with the basics of electricity. In addition, the electrical components that are used to build electronic devices are examined.

Chapter 5 provides the student with an overview of electrical power and the power industry. The student is exposed to topics including the generation and supply of electrical power to the end user.

Chapter 6 introduces the student to the concepts involved in basic circuit analysis. They are first presented with the basics of resistors in series and parallel and taught how to find the equivalent resistances of these configurations. They then learn about voltage and current dividers and Kirchoff's voltage and current laws. Finally, the student is presented with two approaches to more complicated circuit analysis. The basics of both mesh and nodal analysis are presented.

The third unit of the text is designed to provide the student with a background in important computer engineering concepts. Its two chapters provide the student with basic concepts in the areas of digital logic and computer organization.

Chapter 7 presents the basic concepts of digital logic. The student is introduced to the basic logic gates, truth tables and Boolean expressions. They are taught how to analyze a logic circuit to create truth tables and how to build a logic circuit that satisfies a given set of conditions. Finally, students are taught the basics of logic simplification and introduced to the practical considerations involved in the construction of a logic circuit from physical components.

Chapter 8 continues with the student's introduction to computer engineering topics by describing the organization of a typical computer. The various functional components that make up a computer are described and the student is introduced to the way in which these components work together to perform the computer's basic functions.

The fourth unit covers the areas of digital signal processing, image processing, and communication. The four chapters in this unit provide the student with an overview of DSP, sound and image processing, communication theory, and information coding.

Chapter 9 is designed to provide the student with a basic introduction to the area of digital signal processing. The student is presented with an introduction to signals and systems. In addition, some of the basic operations involved in basic signal processing are introduced on a limited scale.

Chapter 10 provides an application of digital signal processing to the area of sound. The concept of audio signals is introduced, and several applications of digital signal processing within this area are presented.

Chapter 11 continues the examination of digital signal processing applications by looking at two-dimensional signals or images. The basics of photography and human vision are presented to show the importance of analog signals. Digital equivalents are given and the concepts of filtering, edge detection, and noise removal are explored.

Chapter 12 deals with the concepts involved in communication. The five parts of a basic communication system are examined. In addition, this chapter also explores the concept of encoding information for ease of transmission and the detection or correction of errors. Several coding techniques are presented.

The fifth and final unit provides several laboratory tutorials (modules), exercises, and design problems written to provide students with an introduction to several concepts and tools they will be utilizing throughout their education in engineering.

Chapter 13 is a self contained Module that introduces students to the MATLAB software package. Topics covered include MATLAB's basic commands, functions, plotting, and MATLAB M-Files

Chapter 14 introduces students to the CAD software package Multisim. Several topics are included such as voltage, current, power, and resistance measurements. This material can be used as a precursor to having the students make real measurements in the laboratory.

Chapter 15 introduces students to serial communications and the RS232 standard utilizing a personal computer and the Windows communication software HyperTerminal.

Chapter 16 covers the topic of operational amplifiers. It provides students with enough information to use op amps as building blocks in simple circuits. Several basic op amp circuits are discussed including the inverting, non-inverting, current to voltage, and summing amplifier configurations.

Chapter 17 introduces basic diodes. It introduces the general purpose silicon diode, light emitting diode (LED), and photodiode. In section 2 (optional) the characterization of semiconductor materials and a more detailed explanation of the pn junction are provided.

Chapter 18 introduces the concepts needed to understand the basic behavior of light and how fiber optic cable acts as a waveguide. The topics of light reflection, refraction, and total internal reflection are discussed.

Chapter 19 introduces students to the concept of PCB (Printed Circuit Board) design utilizing the software package ExpressPCB, A package available for free download at the ExpressPCB website. The software is introduced by having the student enter a simple predefined board design.

Finally, Chapter 20 contains several exercises, and an example design exercise is also presented. The exercises have been designed to provide students with practice applying the material presented in Chapters 13 through 19.

The goal of each of the chapters in this text is not to give the student an in-depth understanding of the topics that they cover, but rather to expose the student to the basic ideas and concepts at a level that their background allows.

Usage

The organization of this text allows instructors to present the material in a variety of different ways. Each of the chapters is self-contained and can stand alone should an instructor desire to present it in such a way. The recommended approach to this text is to present the first three chapters (Unit 1) first. After the students have the required math background, the remaining chapters can be approached in virtually any order.

About the Authors

Dr. Reza Adhami

Dr. Reza Adhami serves as both a professor and the Chair of the Electrical and Computer Engineering Department at The University of Alabama in Huntsville (UAH). He has been a member of the electrical engineering faculty at UAH since 1983. During that time he has held many positions including Instructor, Associate Professor, Assistant Professor, Professor, Chair of the department, and Chair of the Inter-Campus Program Coordinating Board for the Shared Ph.D. degree in Computer Engineering at the University of Alabama in Birmingham and The University of Alabama in Huntsville. His research interests include communication systems, signal processing, image processing, biomedical systems, and VLSI. More specifically, his work has involved such topics as wavelet theory and its applications in signal processing, biomedicine and biometrics. As a faculty advisor, he has successfully supervised numerous Masters Theses and Ph.D. dissertations. He has authored/co-authored over 100 technical papers that have been presented and published in many technical journals and conferences. Dr. Adhami has multiple years of industrial experience during which he has designed and implemented numerous hardware/software systems.

Dr. Peter Meenen

Dr. Peter Meenen has more than seven years of experience teaching introductory engineering concepts to students, primarily at the University of Alabama in Huntsville. He obtained a Bachelor of Science degree from Berry College in the areas of Physics and Computer Science, and a Master of Science and Ph.D. from the University of Alabama in Huntsville in the area of Electrical Engineering with an emphasis in signal processing and pattern recognition. He currently works in industry as a senior engineer where he focuses on developing technology and products in the areas of medical imaging and biometric security.

Mr. Dennis Hite

Mr. Dennis Hite has more than 10 years of experience managing and teaching introductory physics and engineering laboratories. He obtained a Bachelor of Science degree from Purdue University in Physics, and a Master of Science in Engineering with an option in Electrical Engineering from the University of Alabama in Huntsville. He currently holds the position of Lecturer/Information Technology Specialist at the University of Alabama in Huntsville.

Table of Contents

CHAPTER 1. NUMERIC BASES	3
1.1 Introduction to Numeric Bases.....	3
1.2 Working With Arbitrary Bases.....	5
1.2.1 Counting in an Arbitrary Base.....	5
1.2.2 Converting an Arbitrary Base to Decimal	6
1.2.3 Converting Decimal to an Arbitrary Base	8
1.3 Operations in Common Bases.....	12
1.3.1 Binary (Base 2) Numbers	12
1.3.2 Octal (Base 8) Numbers	17
1.3.3 Hexadecimal (Base 16) Numbers.....	21
1.4 Bases that are Powers of Other Bases.....	25
1.5 Simple Addition in Other Bases.....	29
1.6 Chapter Summary.....	31
1.7 Review Exercises.....	33
CHAPTER 2. COMPLEX NUMBERS.....	39
2.1 Introduction to Complex Numbers	39
2.1.1 Background.....	40
2.1.2 What is a Complex Number?.....	41
2.1.3 Why Worry About Complex Numbers?.....	43
2.2 Representing Complex Numbers.....	43
2.2.1 The Complex Plane	45
2.2.2 Complex Numbers in Rectangular Coordinates.....	45
2.2.3 Complex Numbers in Polar Coordinates	46
2.2.4 Converting Between Coordinate Representations	48
2.2.5 Some Related Useful Equations.....	54
2.3 Working With Complex Numbers.....	57
2.3.1 The Complex Conjugate	57
2.3.2 Addition	58
2.3.3 Subtraction.....	59
2.3.4 Multiplication	60
2.3.5 Division	64
2.3.6 Expression Simplification.....	66
2.4 Chapter Summary.....	69
2.5 Review Exercises.....	72

Table of Contents

CHAPTER 3. VECTORS AND MATRICES..... 77

3.1 Introduction 78

3.2 Introduction to Vectors 78

3.3 Working With Vectors 80

 3.3.1 Basic Concepts 80

 3.3.2 Vector Addition 82

 3.3.3 Vector Subtraction 85

 3.3.4 Scalar Multiplication 88

 3.3.5 The Scalar (Dot) Product 88

 3.3.6 The Cross Product 92

3.4 Introduction to Matrices 96

3.5 Matrix Arithmetic 98

 3.5.1 Basic Concepts 98

 3.5.2 Matrix Addition 101

 3.5.3 Matrix Subtraction 103

 3.5.4 Scalar Multiplication 104

 3.5.5 Matrix Multiplication 106

3.6 Other Matrix Operations 110

 3.6.1 Transpose 110

 3.6.2 Determinant 112

 3.6.3 Minors 117

 3.6.4 Cofactors 118

 3.6.5 Matrix Inverse 120

 3.6.6 Singular and Non-Singular Matrices 128

3.7 Application of the Matrix Inverse – Systems of Linear Equations 128

 3.7.1 Introduction to Systems of Linear Equations 129

 3.7.2 Solving Systems of Equations with the Matrix Inverse 135

 3.7.3 Electrical Engineering Application – Circuit Analysis 139

3.8 Chapter Summary 143

3.9 Review Exercises 147

CHAPTER 4. ELECTRICAL ENGINEERING CONCEPTS..... 155

4.1 The International System of Units 156

4.2 The Basics of Electricity 157

 4.2.1 Electrical Charge 158

 4.2.2 Electrical Forces and Fields 161

 4.2.3 Electric Current 167

Table of Contents

4.2.4	Electrical Conductance	169
4.2.5	Resistance	171
4.2.6	Capacitance	174
4.2.7	Inductance	180
4.2.8	Power	187
4.2.9	Alternating Current.....	188
4.2.10	Electrical Circuits	190
4.3	Electrical Components	191
4.3.1	Sources.....	191
4.3.2	Switches	193
4.3.3	Resistors.....	195
4.3.4	Capacitors.....	198
4.3.5	Inductors	199
4.3.6	Transformers	199
4.3.7	Diodes.....	201
4.3.8	Transistors	203
4.3.9	Integrated Circuits.....	205
4.4	Measurement Devices	205
4.4.1	Measurement Errors	206
4.4.2	Ammeters	210
4.4.3	Voltmeters.....	211
4.4.4	Ohmmeters	211
4.4.5	Multimeters	212
4.4.6	Oscilloscopes.....	213
4.5	Practical Example – AC to DC Conversion.....	213
4.6	Chapter Summary.....	216
4.7	Review Exercises.....	221
CHAPTER 5. ELECTRIC POWER		227
5.1	Introduction	227
5.2	A Brief History of Electrical Power	227
5.3	Layout of a Typical Electrical Power System.....	230
5.3.1	Generation	231
5.3.2	Transmission	233
5.3.3	Sub-transmission	234
5.3.4	Distribution.....	235
5.3.5	Connection to the End User	236
5.3.6	End User Internal Distribution.....	238
5.4	Chapter Summary.....	239
5.5	Review Exercises.....	241

Table of Contents

CHAPTER 6. BASIC CIRCUIT ANALYSIS..... 243

6.1 Introduction 243

6.2 Basic Concepts 244

6.2.1 Ohm’s Law 244

6.2.2 Watt’s Law 246

6.2.3 Resistances in Series 250

6.2.4 Voltage Division 253

6.2.5 Kirchhoff’s Voltage Law 255

6.2.6 Resistances in Parallel 257

6.2.7 Current Division 264

6.2.8 Kirchhoff’s Current Law 268

6.3 Circuit Analysis Methods..... 270

6.3.1 Circuit Simplification 271

6.3.2 Mesh Analysis..... 274

6.3.3 Nodal Analysis 278

6.4 Chapter Summary 282

6.5 Review Exercises..... 284

CHAPTER 7. DIGITAL LOGIC..... 295

7.1 Introduction to Digital Logic 295

7.2 Truth Tables 296

7.3 Boolean Expressions 297

7.4 Basic Logic Gates 297

7.4.1 NOT Gates..... 297

7.4.2 AND Gates..... 298

7.4.3 OR Gates..... 299

7.4.4 NAND Gates 300

7.4.5 NOR Gates 301

7.4.6 XOR Gates 302

7.5 Combinations of Logic Gates..... 303

7.5.1 Finding a Truth Table by Circuit Tracing 303

7.5.2 Finding a Boolean Expression for Gate Combinations..... 306

7.5.3 Converting a Boolean Expression into a Truth Table..... 309

7.5.4 Putting the Gates Together – Creating a Useful Logic Circuit 312

7.6 Simplification..... 316

7.6.1 Boolean Algebra 317

7.6.2 Karnaugh Maps 320

Table of Contents

7.7	Logic Word Problems.....	323
7.8	Practical Considerations	327
7.9	Chapter Summary.....	330
7.10	Exercises.....	332

CHAPTER 8. COMPUTER ORGANIZATION AND ARCHITECTURE 337

8.1	Introduction	337
8.2	Basic Computer Organization.....	338
8.2.1	The Power Supply	338
8.2.2	The Clock.....	339
8.2.3	The Central Processing Unit (CPU).....	339
8.2.4	The Bus	345
8.2.5	Memory and Storage	347
8.2.6	Input / Output Devices	350
8.3	Computer Operating Systems	351
8.4	Chapter Review	352
8.5	Review Exercises.....	354

CHAPTER 9. DIGITAL SIGNAL PROCESSING..... 359

9.1	Introduction and Overview	360
9.1.1	What is a Signal?.....	360
9.1.2	Classification of Signals.....	360
9.1.3	What is Digital Signal Processing	366
9.2	Introduction to Some Special Signals	369
9.2.1	The Unit Impulse Function.....	369
9.2.2	The Unit Step Function	370
9.3	Plotting Discrete-Time Signals.....	371
9.3.1	Time Shifting.....	371
9.3.2	Time Scaling	372
9.3.3	Reversal.....	375
9.3.4	Scaling	376
9.3.5	Signal Addition.....	377
9.3.6	Signal Multiplication.....	378
9.4	Working with Signals.....	380
9.4.1	Signal Summation.....	380
9.4.2	Signal Energy	383
9.4.3	Signal Power.....	384

Table of Contents

9.4.4	The Geometric Series.....	386
9.5	An Introduction to Systems.....	389
9.5.1	What is a System?.....	389
9.5.2	Classification of Systems.....	390
9.6	Working with Systems.....	395
9.6.1	Determining a System's Output – Convolution.....	395
9.6.2	Sequence Correlation.....	406
9.6.3	Describing a System – Difference Equations.....	411
9.7	Filters and Filtering.....	416
9.8	Chapter Summary.....	419
9.9	Review Exercises.....	421
CHAPTER 10. APPLIED DSP – SOUND PROCESSING.....		427
10.1	Introduction.....	427
10.2	The Basics of Sound.....	428
10.2.1	What is sound?.....	428
10.2.2	How do we hear sounds?.....	431
10.2.3	Sources of Sound.....	433
10.3	Preparing Audio Signals for Processing – Digitizing Sound.....	437
10.4	Working with Sound - Applications.....	441
10.4.1	Finding the Fundamental Frequency of an Audio Signal.....	441
10.4.2	Modeling Echo.....	443
10.4.3	Generating Audio Tones – Touch Tone Phones.....	447
10.4.4	Identifying Sounds.....	449
10.5	Chapter Summary.....	452
10.6	Review Exercises.....	454
CHAPTER 11. APPLIED DSP – IMAGE PROCESSING.....		457
11.1	Introduction.....	458
11.2	Analog Image Processing.....	458
11.2.1	Analog Photography.....	458
11.2.2	Human Sight.....	460
11.3	Digital Images.....	462
11.3.1	Obtaining Digital Images.....	464
11.3.2	Grayscale Images.....	467

Table of Contents

11.3.3	Dealing with Color	468
11.3.4	Ways of Describing Color – Color Models	470
11.4	Basic Digital Image Manipulation.....	475
11.4.1	Horizontal and Vertical Flipping	476
11.4.2	Cut and Paste	478
11.4.3	Rotation.....	481
11.4.4	Resizing Images	483
11.5	Image Filtering and Enhancement.....	485
11.5.1	How does filtering work?.....	485
11.5.2	Low Pass Filters.....	490
11.5.3	High Pass Filters.....	492
11.5.4	Edge Detection	493
11.6	Noise and Image Restoration	495
11.6.1	Some Common Types of Noise.....	496
11.6.2	Removing Noise.....	497
11.7	Image Compression.....	500
11.7.1	Lossless Compression.....	502
11.7.2	Lossy Compression	504
11.8	Images in Motion - Video	507
11.8.1	How Video Works	507
11.8.2	Video Compression.....	509
11.9	Chapter Summary.....	511
11.10	Review Exercises.....	513
CHAPTER 12. COMMUNICATION AND CODING		517
12.1	Introduction to Communication	518
12.2	The Parts of a Communication System	518
12.3	The Source	519
12.4	The Transmitter.....	520
12.4.1	Modulation	520
12.4.2	Why We Modulate Signals	520
12.4.3	Types of Modulation	523
12.4.4	Classification of Communication Systems by Modulation Scheme.....	526
12.5	The Channel	527
12.5.1	Common Channel Types.....	527
12.5.2	Classification of Communication Systems by Channel Type	528
12.5.3	Allocating the Channel.....	529
12.5.4	Dealing with Channel Noise	532

Table of Contents

12.6	The Receiver.....	533
12.6.1	Demodulation.....	533
12.7	The Destination.....	533
12.8	Introduction to Coding.....	534
12.9	Basic Numeric Coding Schemes	534
12.9.1	Straight Binary Coding.....	534
12.9.2	Binary Coded Decimal.....	535
12.10	Basic Alphanumeric Encoding Schemes	538
12.10.1	The ASCII Code.....	538
12.10.2	Huffman Coding.....	541
12.11	Dealing with Errors.....	547
12.11.1	Parity Error Detection	547
12.11.2	Two-Dimensional Parity.....	549
12.11.3	Cyclic Redundancy Check – CRC.....	552
12.12	Chapter Summary.....	554
12.13	Review Exercises.....	557
CHAPTER 13. Module 1 – MATLAB.....		567
13.1	Basic Math Functions and Matrices.....	567
13.2	Plotting in MATLAB.....	585
13.3	MATLAB M-Files.....	601
CHAPTER 14. Module 2 – Multisim		607
14.1	Multisim CAD Design and Electric Measurements	607
14.2	Digital Design.....	619
CHAPTER 15. Module 3 – Serial Communications		625
15.1	Serial Communications.....	625
15.2	HyperTerminal	632
CHAPTER 16. Module 4 – Operational Amplifiers (Op Amps)		637
16.1	Operational Amplifiers.....	637
16.2	Operational Amplifier Circuits	640

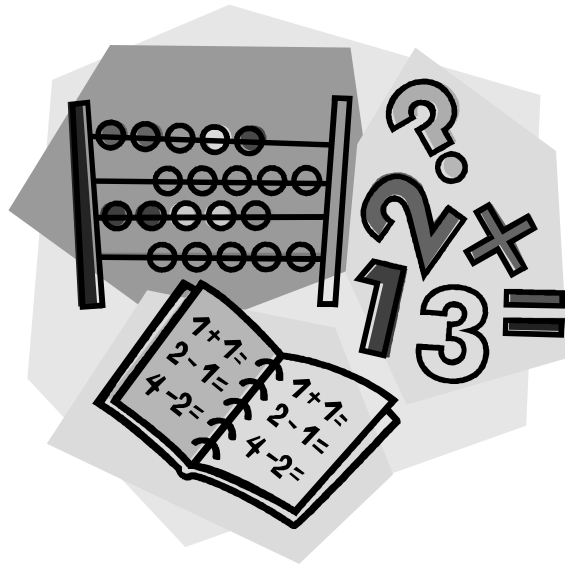
Table of Contents

CHAPTER 17. Module 5 – Light Emitting and Photodiodes	647
17.1 Light Emitting and Photodiodes	647
17.2 PN Junctions	654
CHAPTER 18. Module 6 – Fiber Optic Communications Link.....	661
18.1 Fiber Optic Communications Link	661
18.2 Fiber Optic Cables and Connectors	665
CHAPTER 19. Module 7 – Printed Circuit Board Design	667
19.1 Printed Circuit Board Design	667
19.2 Simple Board Design	670
CHAPTER 20. Exercises	673
20.1 Exercise 1 – MATLAB Basics	673
20.2 Exercise 2 – MATLAB Plotting and M-Files	674
20.3 Exercise 3 – Multisim Design.....	675
20.4 Exercise 4 – RS232 Point-to-Point Communications	677
20.5 Exercise 5 – Op Amp Circuits.....	679
20.6 Exercise 6 – Diodes: LEDs and Photodiodes	681
20.7 Design Exercise – RS232-Fiber Point-to-Point Communications Link	683
Appendices	691
Appendix A - Resistor Color Code	691
Appendix B - Average (DC) and Root-Mean-Square (RMS) Values.....	692
Appendix C - Optimization of Power Transfer	695
Appendix D - Adjustable Voltage Divider.....	798
Appendix E - Viewing RS232 Signals with an Oscilloscope.....	700
Appendix F - PN Junction Bias Voltage.....	701
Appendix G - Indices of Refraction	703
Appendix H - American Standard Code for Information Interchange (ASCII) Table	704
Appendix I - RC Low-Pass Filter.....	705

Table of Contents

References	709
Acknowledgements	711
Index	713

UNIT 1



Basic Mathematics

Chapter 1. Numeric Bases

Chapter Outline

CHAPTER 1. NUMERIC BASES	3
1.1 Introduction to Numeric Bases	3
1.2 Working With Arbitrary Bases	5
1.2.1 Counting in an Arbitrary Base.....	5
1.2.2 Converting an Arbitrary Base to Decimal	6
1.2.3 Converting Decimal to an Arbitrary Base	8
1.3 Operations in Common Bases	12
1.3.1 Binary (Base 2) Numbers	12
1.3.2 Octal (Base 8) Numbers	17
1.3.3 Hexadecimal (Base 16) Numbers	21
1.4 Bases that are Powers of Other Bases.....	25
1.5 Simple Addition in Other Bases	29
1.6 Chapter Summary	31
1.7 Review Exercises.....	33

1.1 Introduction to Numeric Bases

No matter what your occupation, you will encounter and work with numbers on a regular basis. Whether you are making change at a cash register, balancing your checkbook, or calculating power consumption of an electrical circuit, you must have a basic understanding of how numbers work. Take for example, the simple task of counting from 0 to 100. While this is a trivial task, you must know something about numbers to accomplish it. You begin counting using a single placeholder, the “ones” place. You know that when you reach the largest digit that can fit in this place (in this case 9) you must add an additional placeholder to continue. Thus, to proceed beyond 9, you add the “tens” place. One is added to this new place, and the original “ones” place is reset to zero. By doing so, you can proceed from 9 to 10. Likewise, each time you reach a 9 in the “ones” place, the next number is determined by incrementing the “tens” place and resetting the “ones” place. Following this procedure, you can count all the way up to 99. At this point, an additional placeholder, the “hundreds” place is used. By extending this procedure, it is possible to count to numbers far larger than 100.

While this example may seem obvious, it illustrates an important point about numbers. As you saw, there was a particular number that, to be properly represented required the addition of an additional placeholder. In this example we found that each time a placeholder reached the

number 10, we had to add a new placeholder to our number in order to represent it. When the “ones” place reached 10 we had to add the “tens” place, and when the “tens” place reached 10, we had to add the “hundreds” place. The number that, once reached, requires this addition of a new placeholder is called the radix. So, from our example, we find that the numbers we are used to working with have a radix of 10. Knowing the radix gives us important information about the number system that we are using, thus, the radix number is often referred to as the base of the number system. In other words, we typically work with numbers using a radix of 10, so we can say we use a base 10 number system. This base 10 system is often referred to as the decimal number system (from the Latin word “Decima” which means a tenth).

The decimal number system is not the only one used, however. Many applications, especially in electrical engineering and other technical fields, require the use of number systems based on a different radix. One good example is the binary, or base 2, number system. It is commonly used in the areas of digital logic and computers. These areas deal with signals that can have only two possible states, on or off. Thus a number system with a radix of two is perfectly suited for describing events in these systems.

Before we look too closely at the use of other bases, let’s get a better understanding of the mechanics of numbers in the decimal system. In order to see exactly how decimal numbers work, let’s look at an example of a typical decimal number. In this case, let’s use the decimal number $(1279.875)_{10}$. You may have noticed that this number is not written quite the way you might expect. The parentheses and subscript 10 is part of a notation method that is often used when dealing with numbers of different bases. It helps us avoid confusion by keeping track of what base each number is in. If we did not specify the base each number is in, it would be easy to get confused, and that could lead to unwanted errors. Now that we are comfortable with this notation, let’s look at what this decimal number is really telling us. Figure 1.1, shown below, breaks the number down into its respective places.

	1	2	7	9	.	8	7	5
	10^3	10^2	10^1	10^0		10^{-1}	10^{-2}	10^{-3}
Thousands	1000							0.001 Thousandths
Hundreds	100							0.01 Hundredths
Tens	10							0.1 Tenths
Ones	1							

Figure 1.1. An example of a typical decimal (Base 10) number.

As you can see, by breaking the number down, we find that each of the digits gives us important information. The numeric value in each of the placeholders indicates the degree to which its respective power of the base is represented in the total number. In this example, it is easy to see that there are 1×10^3 or 1 “thousand,” 2×10^2 or 2 “hundreds,” 7×10^1 or 7 “tens,” 9×10^0 or 9 “ones,” 8×10^{-1} or 8 “tenths,” 7×10^{-2} or 7 “hundredths,” and 5×10^{-3} or 5 “thousandths” in the number. As a result the final number can be thought of as $(1 \times 10^3) + (2 \times 10^2) + (7 \times 10^1) + (9 \times 10^0) + (8 \times 10^{-1}) + (7 \times 10^{-2}) + (5 \times 10^{-3})$ or $1000 + 200 + 70 + 9 + 0.8 + 0.07 + 0.005 = 1279.875$. While this may seem obvious when working with decimal numbers, the same concept can be used to work with numbers of any base. To better understand this, let’s first look at how this works for an arbitrary base b . We will then examine numbers in some of the more commonly used bases.

1.2 Working With Arbitrary Bases

While working with numbers in a specific base is useful, it is far more beneficial to be able to work with numbers in any base. Since you never know when you may encounter numbers in an unfamiliar base, it is important to have general tools that can be adapted to work with any base you choose. In this section, we will explore the basics of bases. We will learn how counting works in different bases, and we will learn how to convert numbers between an arbitrary base and decimal.

1.2.1 Counting in an Arbitrary Base

Once we have determined what base we are working with, let's call it b , we automatically know two important things about our number. First, we know that each digit or placeholder in the number is going to represent a particular power of the base, and second, there are exactly b values that can be used in each placeholder. These values will range from 0 to $(b - 1)$. As a result, when we count in a base b number system, each placeholder will begin at 0 and increment up to $b - 1$. Once a placeholder has reached $b - 1$, a new placeholder must be added to count beyond that point.

To help clarify this concept, let's look at an example.

Example 1.1

Problem: Count to $(20)_{10}$ in base 5

Solution: Since we have been given a base to work in, we can assign a value to our arbitrary base b . In this case let $b = 5$. Since we now know that b is equal to 5, we know that each digit of any number represented in this base gives us an indication of the degree to which each power of five is represented in the number as a whole. We also know that each digit of the number will range from 0 to $(5 - 1)$ or 4. So, let's count to $(20)_{10}$ in base 5.

Decimal	Base 5
0	0
1	1
2	2
3	3
4	4

As you can see we have reached the value of $b - 1$, which in this case is 4. Since our radix is 5, we know that when a placeholder reaches 5 a new placeholder must be added to represent the number. Let's add a placeholder and continue.

Decimal	Base 5
5	10
6	11
7	12
8	13
9	14
10	20

At this point, we have reached $(20)_5$. However, this does not mean we have finished counting. If we look at the table, we will see that $(20)_5$ is equivalent to only $(10)_{10}$. We are only half-way there. Let's count the rest of the way now.

Decimal	Base 5
11	21
12	22
13	23
14	24
15	30
16	31
17	32
18	33
19	34
20	40

As you can see, we have now reached $(20)_{10}$ in the base five number system. From the chart we see that $(40)_5$ is equal to $(20)_{10}$.

This example gives us a good idea of how counting in a base other than decimal works. At this point you may ask, is there any way to tell what a number in some other base b will equal in decimal without building a chart and counting? The answer is yes.

1.2.2 Converting an Arbitrary Base to Decimal

The process used to determine the decimal equivalent of a number expressed in a different base b is actually very simple. The key is to remember the function of each of the placeholders in the number. Each placeholder determines to what degree a certain power of the base is represented in the final number. In addition, each placeholder to the left of the radix point (note that this is what a decimal point is called in number systems that are not base 10) represents an increasing positive power of the base, and that each placeholder to the right of the radix point represents an increasing negative power of the base. Let's take a look at what this would look like for a general base b .

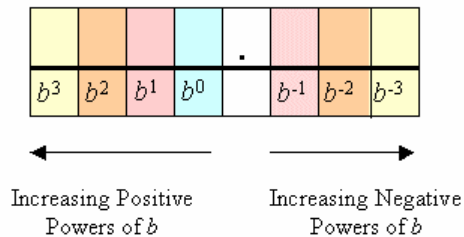


Figure 1.2. An illustration of the relationship between each placeholder and its respective power of the base.

Since each placeholder represents a specific power of the base, we can obtain the decimal value represented by a given placeholder by simply multiplying the value in that placeholder by the value obtained by raising the base to the appropriate power. If we calculate this value for each placeholder and sum the results, we will arrive at the decimal equivalent of the number represented in base b . To illustrate this, let's look at base 5 once again. Figure 1.3 displays the powers associated with each placeholder in the base 5 system.

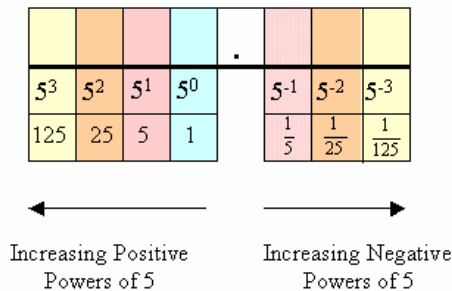


Figure 1.3 A representation of the powers of 5 associated with the placeholders in a base 5 number.

Continuing with our base 5 example, let's find the equivalent decimal number for $(13412.213)_5$.

Example 1.2

Problem: Find the equivalent decimal number for $(13412.213)_5$.

Solution: Our first step in converting this number to its decimal equivalent is to multiply the value in each placeholder by its respective power of the base, in this case 5. The following table illustrates this process.

Placeholder Value	Associated Power of the Base	Resulting Product
1	$5^4=625$	$1 \times 625 = 625$
3	$5^3=125$	$3 \times 125 = 375$
4	$5^2=25$	$4 \times 25 = 100$
1	$5^1=5$	$1 \times 5 = 5$
2	$5^0=1$	$2 \times 1 = 2$
.	Radix Pt.	
2	$5^{-1}=0.2$	$2 \times 0.2 = 0.4$
1	$5^{-2}=0.04$	$1 \times 0.04 = 0.04$
3	$5^{-3}=0.008$	$3 \times 0.008 = 0.024$

Now that we know the decimal value associated with each placeholder, all we have to do is add these values together to arrive at our answer.

$$\begin{array}{r}
 625.000 \\
 375.000 \\
 100.000 \\
 5.000 \\
 2.000 \\
 0.400 \\
 0.040 \\
 + 0.024 \\
 \hline
 1107.464
 \end{array}$$

So, now we know that $(13412.213)_5$ is equivalent to $(1107.464)_{10}$.

In the preceding example, we found the decimal equivalent of a base 5 number. The method we used in this example will work to convert a number in any base to its decimal equivalent. Simply multiply the value in each placeholder by its respective power of the base and add the results.

MATLAB: MATLAB has a function that allows for the conversion of an arbitrary base to its decimal equivalent number. This function has the following usage syntax.

BASE2DEC(S,B)

Where S is a string containing the number in base B, and B is the base that the number is in. Keep in mind that this function only converts positive integer values. It will not work on negative numbers or numbers containing fractional parts.

Example: Base2Dec('13412',5) will return 1107 which is the decimal equivalent of the specified base 5 number.

1.2.3 Converting Decimal to an Arbitrary Base

Now that we know how to find the decimal equivalent of a number given in another base, our next goal is to learn how to work in the other direction. We want to be able to represent a given decimal number in another base. The procedure required to perform this conversion is slightly more complicated than the conversion to decimal, but it is still not too difficult. Unlike the process of converting a given base to decimal, the reverse process requires separate processing for the whole and fractional parts of the number.

Let's look at the whole number part first. The easiest way to determine the representation of the whole number portion of a decimal number in a different base is by successively dividing the decimal number by the radix of the new base and keeping track of the remainder. This sounds difficult at first, but is actually quite easy. If we wish to perform the conversion of a whole number from decimal to base b , we must divide our number by b . The remainder of this division will be the rightmost digit of our base b number. Next, we divide the quotient from the previous division by b . The remainder from this division is the next digit in our base b number. This division process is repeated until the quotient is zero. At this point we have successfully converted our decimal number to the new base. Figure 1.4 illustrates this process.

$$\begin{array}{r}
 0 \\
 \hline
 b \overline{) Q_3} \\
 \hline
 b \overline{) Q_2} \\
 \hline
 b \overline{) Q_1} \\
 \hline
 b \overline{) Decimal}
 \end{array}
 \begin{array}{l}
 R_4 \\
 R_3 \\
 R_2 \\
 R_1
 \end{array}$$

$$(\text{Decimal})_{10} \longrightarrow (R_4 R_3 R_2 R_1)_b$$

Figure 1.4. Division method for base conversion.

Having completed the conversion of the whole number portion of the decimal number, we can now focus on the fractional portion. While the whole number portion was converted using division, the fractional portion of the number is converted using multiplication. In this case, the fractional part of the number is multiplied by the radix of the new base. After the multiplication we record the whole number portion of the result. Then, if the fractional part of the number is different from any of the previously encountered fractional parts, we continue to multiply and track the whole number portion. It should be noted that the multiplication at each stage is applied to only the fractional part of the number. Any whole number portion from previous multiplications is ignored. The process comes to an end when either, the fractional portion of the number becomes zero or, a duplicate fractional portion is encountered. If the fractional portion of the number becomes zero, the conversion to the new base was successful and the resulting number can be represented with a finite number of placeholders. If a duplicate fractional portion is encountered, the representation in the new base requires a repeating number (Infinite Placeholders). In this case, all the numbers between the first and second occurrence of the duplicate fractional part will repeat. Figure 1.5 illustrates the procedure.

$$\begin{array}{l}
 0.dddd_1 \\
 \times \quad b \\
 \hline
 w_1.dddd_2 \\
 \times \quad b \\
 \hline
 w_2.dddd_3 \\
 \times \quad b \\
 \hline
 w_3.0000
 \end{array}$$

$$(0.dddd)_{10} \longrightarrow (0.w_1 w_2 w_3)_b$$

Figure 1.5. An illustration of the conversion of fractional numbers to a new base.

The best way to truly understand the conversion from decimal to another base is by actually looking at an example. First, let's look at the number $(1107.464)_{10}$. This is the number that we arrived at after our conversion from base 5 to decimal in the previous example. If we use the