GRAVITY
Galileo to Einstein and Back

Newtonian Force, Slave or Master?

H.R. Harrison

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Prologue

The route to understanding gravity really started with the insight of Galileo Galilei, who stated that all bodies, in the absence of air drag, fall to Earth with a velocity which increases linearly with time. The astronomical observations of Tycho Brahe led to Johannes Kepler's laws of planetary motion, which describe the paths of the planets around the Sun as being elliptical. The genius of Isaac Newton was to see that the phenomena were related and , then, to devise his laws of motion and the law of gravity such that any motion could be predicted with astonishing accuracy. All was well until small differences from the Newtonian theory were observed. The main ones being the precession of the perihelion of Mercury (i.e. the ellipse itself slowly rotates) and the deflection of starlight passing close to the Sun. The details of these observations will be discussed in later chapters.

In order to explain these variations, Albert Einstein developed his General Theory of Relativity, which involves the concept of a curved space-time continuum. This replaces the concept of gravitational force devised by Newton. The notion of curved space-time is not a simple concept to grasp, but it is mathematically very elegant and, so far, has not been faulted. The question which is raised is this: *Is it necessary to invent the concept of space time curvature to replace the concept of force when it is possible to extend the special theory of relativity?*

The problem stems from the nature of force. Is force one of nature's fundamental quantities, or is it purely the invention of man? The argument has been discussed for three centuries with both ideas finding favour. Force is often ill defined with the writer and the reader using their own intuitive definitions. The aim in this book is retrace the steps from Galileo and see if a conceptually easier path can be taken that will enable us to calculate all the known observations. *Is force the master or the slave in dynamics?*

The modification suggested in this book, which will extend the theory of gravity, is similar to the modification made to electrostatics by Einstein's Special Theory of Relativity, which gives an explanation of magnetism. By this means,

Gravitomagnetics is introduced and yields the same results for the above mentioned variations to Newtonian mechanics, as does the General Theory of Relativity. However, it is not identical in all other aspects.

My views on dynamics are the result of over fifty years of study, application in aircraft design, research, university lecturing, and writing text books. Newtonian force was paramount in all these areas. Latterly, I have extended my study into Special Relativity and, then, on to General Relativity. At this point, I found it necessary to re-evaluate the fundamentals of dynamics. My long standing understanding was that mass, length, and time were primary and axiomatic and that force was secondary and defined. This view, taken by H. R. Hertz and others, makes one of the starting points of General Relativity unnecessary. This point is known as the (Weak) Principle of Equivalence. In one form, it says that "inertial mass is equivalent to gravitational mass." Alternatively, "all bodies fall at the same rate independent of their masses or of the material from which they are made." The first form implies the second, but the second does not require the first. This will, of course, be considered in detail later. Einstein's later statements on the equivalence principle are concerned with the equivalence of all (non-rotating) frames for all experiments in physics. In his own words [21] "...it is impossible to discover by experiment whether a given system of co-ordinates is accelerated, or whether its motion is straight and uniform and the observed effects are due to a gravitational field (this is the equivalence principle of the general theory of relativity)." This statement does not depend on the weak principle.

A reader who has a good knowledge of gravity is advised to read the epilogue first, so that my reasons for restating the fundamentals of dynamics may be apparent. Much of the early chapters will be familiar, but I hope that the presentation will show some features from a different viewpoint. The later chapters deal with the new approach to relativistic gravity.

Chapter 1
Newton and Force

1.1 A brief history of force

Force is one of the first physical notions that confront us. As children, we are aware that to move our bodies or any other object, we have to make an effort and use our muscles. Probably the first type of force of which we are aware is that due to gravity. To raise ourselves from a supine position to an upright one is quite an achievement. Should we lose our balance and fall, we soon become aware of another kind of force, contact force. This time we may well feel pain if our head hits a hard surface. The next manifestation of force is that of sliding friction. If we try to move an object across the floor, we need to push it; when we stop pushing, the object stops. As we increase in strength and stability, we find that it is sometimes easier to move small objects by first lifting them. We now develop a clearer idea of the nature of the force of gravity.

The Ancients believed that bodies only move if they are being pushed or pulled. After it was agreed that the planets moved around the Sun, the belief , then, was that something must be pushing them along their paths. On Earth, attention was now being focused on the way bodies fall. Galileo (1564-1642) finally stated that when a body falls, the distance fallen is proportional to the square of the time. If in one unit of time the body falls one unit of distance, then, in each subsequent unit of time the distance will increase by 3 units, then by 5 units, then by 7 units. That is, the total distance after 4 seconds is 1 + 3 + 5 + 7 = 16 = 4 squared. It follows from this that the speed increase is proportional to time because the speed is the distance travelled in unit time. It was thought by many at that time that the speed of fall was proportional to the distance fallen. This can readily be shown to lead to a time distance relationship which is not tenable (see note 1). It should be mentioned that Leonardo Da Vinci (1452 - 1519) had already stated that he was in favour of the law that gave the velocity of fall as being proportional to time, but did not justify it.

Galileo made another highly significant proposition, and that was that all objects fall at the same rate, without regard to their size or to the material from which they are made. The assumption that is implicit here is that air drag must be negligible.

The Danish astronomer Tycho Brahe (1546-1601) made very detailed measurements of the movements of the planets. These were so accurate that they lead Johannes Kepler (1571-1630) to formulate his three laws of planetary motion. The first says that the paths are elliptical, the second that equal areas are swept out in equal times. The third states that the square of the period of revolution about the Sun is proportional to the cube of the average distance from the Sun.

The period of revolution, T, of a planet having a circular orbit about the Sun is the quotient of the circumference of the circle, radius r, and the orbital speed v. So $T = (2\pi r)/v$.

Now the centripetal acceleration $a = v^2/r$ so that

$$T^2 = \frac{(2\pi)^2 r^2}{v^2} = \frac{(2\pi)^2 r}{a}$$

or

$$a = \frac{(2\pi)^2 r}{T^2} \quad .$$

Kepler's third law states that $T^2 \propto r^3$ hence $a \propto \frac{1}{r^2}$. We conclude that the acceleration of a planet is proportional to the inverse square of the radius and is independent of its mass.

We now, as did Newton, turn our attention to the orbit of the Moon. The centripetal acceleration of the Moon, using the above equation, is 0.00272 m/s^2. The standard measured value of the acceleration of a body falling to Earth at the surface is g = 9.807 m/s^2, and the ratio of these two values is 3606. The ratio of the radius of the orbit of the Moon to the radius of the Earth is 60.33 which when squared is 3640, a variation of only 1%. This gives strong support to the inverse square law. Two extra factors should be taken into account. One is that the effect of the Moon's mass, this is about 1% of the Earth's mass. Second is the effect of the rotation of the Earth, which means that g due to gravity alone is less than 1% bigger than the measured value at a latitude of 45^0.

Starting with Galileo's observations and Kepler's laws of planetary motion, Isaac Newton then calculated the Moon's acceleration and demonstrated his true genius by linking the fall of bodies on Earth and the motion of the planets about the Sun. That an inverse square law was a possibility had already been muted, but no one prior to Newton had been able to show that the elliptical orbits of the planets satisfied the law. Let us now examine Newtonian force.

1.2 Newtonian force

The word "force" is part of everyday speech and is often synonymous with strength, coercion, energy, etc. It relates either to something (or somebody) being compelled to change its state of motion, or it may refer to some innate property of that body. Three hundred years ago, when the foundations of dynamics were being formalised, it was customary to use Latin for scientific writing. The Latin word that was commonly used was *vis,* which generally translates to force but did have other interpretations such as kinetic energy (*vis motrix)*. As a result, different interpretations are, to this day, made or implied. The intention here is to look at these differences.

First, let us look at the Newtonian approach and briefly study its development. The basic axioms are:

1. Space is absolute, and there appears to be a preferred set of axes (frame of reference) for which the laws of dynamics have a simple formulation. (A frame that has constant velocity and no rotation relative to an inertial frame is also an inertial frame.)
2. Time is also absolute and independent of space.
3. Matter is such that if a material of uniform density is used to form bodies, then, the amount of matter is proportional to their volumes. The mass of a body made from a different material can be determined by making use of the laws of motion, which are given later.

The important point here is that the concepts of mass, length, and time are axiomatic, and each is independent of the others and cannot be described in terms of other physical quantities.

Newton's Definitions

Definition *I* is a paraphrase of point 3 above. It also includes the statement that weight is proportional to mass.

Definition *II* is basically that the *quantity of motion* is the product of mass and velocity, i.e. momentum.

Definition *III*, the *vis insita,* is the innate 'force', or inertia, which resists any change in the quantity of motion and is proportional to the mass of the body. This is simply another way of saying that the more massive a body is, then, the more difficult it is to change its state of motion. Here we see that *vis* is used to signify a property of a body rather than a force in the modern sense.

Definition *IV, vis impressa,* is the impressed force or contact force. This is unambiguously a force and is generally the action of one body upon another. In macroscopic terms, this is a short-range force. This force changes a body's state of rest or of uniform straight-line motion.

Definition *V, vis centripetal,* literally the centripetal force. The definition describes that which we would now call a central force, i.e. that force which acts between bodies over finite distances and draws them together (or pushes them apart). Gravity is cited as a typical case. In contrast with definition *IV,* this could be termed a long-range force.

We now come to Newton's laws of motion.

1. A body continues in a state of rest or of uniform motion unless acted upon by a force.
2. The time rate of change of momentum (quantity of motion) is proportional to the force and takes place in the direction of the force.
3. To every action there is an equal and opposite reaction.

The first law is a restatement of definition *III*, sometimes referred to as the law of inertia. For the second law to be a law, it is necessary for force to be an independent physical quantity. The second law then gives the relationship between force and the rate of change of momentum. *In this case, force is the master of dynamics.* It appears that this was in fact Newton's opinion, and it was later supported by D. Bernoulli and Euler.

It is perfectly reasonable to regard the second law simply as the *definition of force*, a quantity defined to aid calculation similar to the way that currency relates to trade. It is possible to construct a system of dynamics that does not involve force, since force is never the final quantity to be determined. However, it is

convenient to use the concept of force in dynamics in the same way that it is convenient to use money to conduct an economy rather than using barter. This concept of force makes *force the slave of dynamics.* H. R. Hertz and d'Alembert supported this view. The third law is a law that can be tested experimentally and is equally applicable whichever view of force we adopt.

In the history of mechanics, both views of force have been expounded, but, provided that we confine our studies to low speed macroscopic dynamics, both views can coexist since the same equations are used. Even in this realm, some imprecise comments are still made.

The most common dilemma arises when discussing circular motion. If a vehicle is in a steady turn in a horizontal plane, then, to ensure that a passenger in the vehicle remains in the same position relative to the vehicle, it is necessary for there to be a contact force between the passenger and the side of the vehicle and/or the seat. The value of this force is readily calculated; it is the centripetal force giving rise to the required centripetal acceleration. However, comments like "the centrifugal force forcing the passenger to the side of the vehicle is real" are still to be seen. Whether this force is real or fictitious is very much a matter of definition. However, neither of the concepts cited above will support such a statement. If a force is required to produce acceleration, then, it is not logical that acceleration should generate a force. For the latter to be acceptable, we need a new concept of force. We would require the definition to be such that the sum of the forces on any body, or group of bodies, shall be zero at all times.

The difficulty arises from the fact that a real (as defined above) long-range body force such as gravity is very similar to the fictitious centrifugal force. The similarity is stronger if we consider straight-line motion. If a contact force produces acceleration in a body which hitherto had a constant velocity, then a similar situation exists if a gravitational force is resisted by a contact force and the motion is that of constant velocity, the state of rest being one possibility. A further example is that of a condition of free fall, which is an accelerating frame of reference, looks very much like an inertial frame without gravity. However, in the case of free fall, a group of bodies initially on parallel equal

velocity paths will change their relative positions. This will be discussed later.

If we regard the centrifugal force as a real force, then, we have a problem with the third law. Returning to the vehicle discussed earlier, the equal but opposite force to the centripetal force on the passenger is the centrifugal force acting on the side of the vehicle. If we imagine that there is a real centrifugal body force, then, where is *its* reaction? One might argue that it is the centripetal force due to the side of the vehicle, but then we see that the resultant force on the passenger is zero. This is a contradiction, since the body is in accelerated motion.

Mass times acceleration can be treated as if it were a force by two methods. One is the notion of the reversed effective force, in which all mass-acceleration terms have their signs reversed and treated as real forces. The problem then looks like an exercise in statics; this is not d'Alembert's Principle (see note 2). The second is to choose non-inertial frames of reference, in which case fictitious forces have to be introduced, so that the sum of all forces is equal to the mass times acceleration relative to the new frame. This approach leads to Coriolis's Theorem (see note 3).

The elusive nature of force is seen when attempts are made to measure force under dynamic situations. For example, if two long steel bars collide axially, then a disturbance will travel in both directions from the impact point at a speed of about 5 000 m/s relative to the unstrained material. It is possible to measure displacement and strain as functions of time, but there is no direct way of measuring stress or force. It is only possible to measure velocity changes and *deduce* the force from the time rate of change of momentum. Alternatively, we could measure strain and assume that this is proportional to the stress. This is only true when dealing with a linearly elastic solid. But how do we know this condition holds under dynamic situations?

At this point, it is advantageous to clarify one of the misconceptions of the nature of force. Frequently, force is associated with the product of mass and acceleration, but it is more fundamental to use the original definition associating force with the time rate of change of momentum. This reverts to the first form when dealing with particles and the motion of the centre of mass of rigid bodies. A rigid body is one which does not change its shape or its volume when forces are applied. This is an

idealisation as a true rigid body does not exist, however it is a useful approximation in a large number of situations. The kinematics of a rigid body requires only six parameters to define its position and orientation: for example, three to locate the centre of mass and three to define the orientation. The other extreme is to have three co-ordinates for every particle in the body. Another method is to treat a body as a continuum, which means an infinite number of parameters, this has mathematical advantages. In the following example we shall consider a steel bar to be a continuum.

Consider two long steel bars impacting co-axially, as shown in Figure 1.1: the left bar moving to the right at a speed v and the right moving to the left at a speed v. The impact velocity will be $2v$ and, by symmetry, the impact point will be stationary. Strain waves will propagate in each bar away from the impact point and it can be shown that the maximum strain and, for a linear elastic material, the stress are proportional to the impact velocity. That is the change in velocity. The rise time for the strain is unimportant provided that no reflections occur from the far ends of the bars. The local acceleration can be exceedingly high as measured impact times are in the order of milliseconds, so that even at impact speed of around 5 m/s accelerations of the order 1 000 m/s^2 are normal.

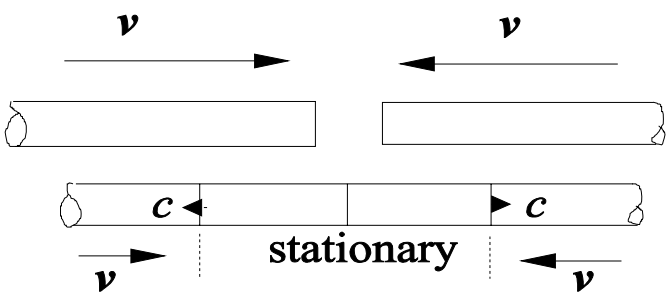

Figure 1.1
Longitudinal impact of two elastic bars

Let us consider this problem in more detail. First, assume that the wave speed, c, is constant. Taking the plane of contact as the reference plane we assume that after a period of time, t, a portion of the bar of length ct will be stationary whilst the remaining part of the bar is still moving with a speed v. If the bar has a density,

ρ, and a cross section area, A, then the change in momentum which occurs in time Δt will be

$\Delta p = \rho A c \Delta t v$ (note that here the mass involved is increasing whilst the velocity involved is constant). So, the (compressive) force, by Newton's definition is

$F = \dfrac{\Delta p}{\Delta t} = \rho A c v$ or, since stress, σ, is (tensile) force per unit area

$\sigma = -\rho c v$.

Now, for a linearly elastic material, stress equals strain times Young's modulus or $\sigma = \varepsilon E$, but the strain which is the change (increase) in length per unit length is $\varepsilon = -\dfrac{v \Delta t}{c \Delta t} = -\dfrac{v}{c}$, so we have

$$\varepsilon = \frac{\sigma}{E} = -\frac{v}{c} \quad \text{or} \quad \sigma = \frac{Ev}{c} = \rho c v, \quad \text{hence} \quad c = \sqrt{\frac{E}{\rho}}$$

We have shown from this simple application of basic definitions that the stress is proportional to the impact velocity and that the wave speed is $\sqrt{E/\rho}$. For many metals, e.g. steel and aluminium, this is approximately 5 000 m/s. So, if the impact duration is only 1 ms, the wave will have travelled 5 m; therefore, for small hard objects, reflection from the far end will occur before the impact is over. In these cases, the notion of a rigid body is reasonable. Obviously, if some form of cushioning is provided, the impact time will increase allowing for more reflections to occur.

It is very difficult for the impact problem considered to apply force equals mass times acceleration because the acceleration occurs for an unknown small number of particles for an unknown very short time; more importantly, the acceleration is irrelevant.

The concept of force plays an important part in the discussion of problems in statics. Here, there is no motion, from which it follows that the resultant force acting on any group of stationary particles is zero. The forces are attributed to several causes. One of the commonest is gravity. Another is associated with changing the shape or volume of a solid body. Analogous with gravity is the

attraction or repulsion between electrically charged bodies. Magnets appear to attract and repel under static conditions, but the work of Oersted, Biot and Savart, Ampere, and Faraday showed that magnetic effects are associated with electric current. Also, the relative movement between conductors and magnets caused interactions. Maxwell then integrated electricity and magnetism with his famous equations. Hertz then generated radio waves and showed that they travelled at the speed of light.

These studies now show that magnetic effects are associated with the movement of charged particles, and Einstein's Special Theory of Relativity gave a solid basis for electromagnetic theory. If moving charges produce measurable effects, then, why not moving mass?

It is now easy to see that the concept of force as devised by Newton is extremely useful. We have the definition of force in its association with rate of change of momentum or, in many cases, mass times acceleration. As a convenient secondary standard we can use weight; from this we can further use elastic deformation to calibrate a spring. So, the concept of force allows us to link together different situations involving weight, deformation and acceleration. In all cases force is just the go-between

1.3 Newtonian dynamics from a different viewpoint

The development of dynamics was based on two very different sets of observations; one was of objects falling to Earth and the other was of the planets orbiting the Sun. In both cases, we have a relatively small object being attracted by a much larger one. Weight and friction dominate the movement of terrestrial objects; whereas, for the heavenly bodies, only position and time were observable. As already mentioned, it was the genius of Isaac Newton that gave us the common link between the two phenomena.

Let us now suppose the observations were made in a very different environment. One could imagine a very small planet or perhaps a large spaceship tumbling in space. Let our observer have a laboratory, which may be orientated relative to the spaceship.

Initially, the experiment consists of projecting small objects within the laboratory and noticing that their trajectories are curved and do not seem to be repeatable. Next, the experimenter uses a

spinning wheel, and after many observations manages to get the laboratory into such a motion, relative to the spacecraft, that the axis of the gyroscope describes a cone. Now, aligning one of the reference axes along the central axis of this cone, it is noticed that objects which are projected parallel to this axis proceed along straight lines. A little more experimentation locates the remaining axes, so that straight-line motion ensues for motion in any direction. This defines a preferred set of axes, for which the motion is as simple as possible. It is simple now to show that any set of axes moving at constant speed, but not rotating, relative to the preferred set would be just as suitable.

Our observer now manufactures two sets of objects: one of lead and the other of steel (say). We state that the amount of matter in each lead object is proportional to its volume. Similarly for steel, but as yet we do not know the relationship between the two. Using objects of one material only, it is not difficult to imagine a set of experiments which would give rise to the conservation of momentum principle. (It can be shown that if the spacecraft is a spherical shell, then, its attraction to objects inside the shell is zero.) However, with the steel bodies in gentle impact situations, the observer would note that, in addition to the conservation of momentum in any direction, the sum of the product of the amount of matter and the squares of their respective speeds (twice the kinetic energy) would remain approximately constant. When using the lead bodies, this sum would rapidly decrease. With the steel objects, it would have been noticed also that the impacts were nearly reversible. That is, any record made of the events would closely obey the rules, even when read backwards in time. One obvious consequence is that the velocity of approach before an impact would equal that of recession.

Armed with these two rules, we can predict the outcome of collisions in two specific cases, (a) if the bodies coalesce and (b) if the impact is reversible. If we now make the assumption that the conservation of momentum is always valid, then, we can use this principle to compare the amount of matter in any two objects made from any material. We can now call this quantity the mass.

Our observer next places two bodies a fixed distance apart and then releases them. At first, the bodies appear to remain fixed in space, but soon the observer realises that the bodies are moving towards each other with increasing speeds. Detailed experiments

will reveal that their *relative* acceleration is inversely proportional to the square of their separation and proportional to the *sum* of their masses.

We can define a position such that the product of the mass of a body and its distance from this point is the same for both bodies. This point is the centre of mass. It follows that the product of the mass and the acceleration towards this point will be the same for both bodies. A little more experimentation will show that the magnitude of this mass times acceleration will be proportional to the *product* of the two masses and inversely proportional to the square of their separation. Now, mass times acceleration is here simply the rate of change of momentum with reference to the centre of mass. We could define this quantity to be force and the particular force here to be a gravity force or weight. This enables us to calculate the acceleration of each body; the resultant being the sum of the effect that each body has on each other. It is noted that the sum of these forces, resolved in any direction, will be zero.

Using this concept of force it is possible to consider a system comprising three bodies X, Y and Z as a system of just two bodies Y and Z and to replace the effect of body X by the forces as defined. The motion is now relative to the centre of mass of the three masses.

During the impact of two bodies, there will be complex movements of the parts of the bodies, but after the impact each body will have suffered the same change in momentum, but in opposite senses. We do not know the precise force at any instant, but we can say that the time integral of the force will be the same for both bodies, but again of opposite sign. This is, of course, the impulse. If the duration of the impact is known, then, a time average of the force is easily found.

The next innovation to be explored is that of the spring. Relatively, light and flexible springs are manufactured. One experiment is to place a spring between two bodies during collision tests. It is observed that momentum is conserved at all times but that kinetic energy is much reduced and falls to a minimum at the instant when the two bodies are moving with the same velocity. After the bodies have separated, the original kinetic energy (half the product of mass times speed squared) is almost wholly recovered. It would seem reasonable to consider that the

loss of kinetic energy during the impact is temporarily stored in the spring. Further experiments will show that the strain or potential energy stored is roughly proportional to the square of the deflection of the given spring. We could now place the spring between two bodies and study the relationship between the gravitational force and the spring deflection. Here we will find an approximately linear relationship. We now have a secondary standard for measuring force, and the usefulness of the concept of force is further established. But note that it has been introduced as a convenience and not as a necessity.

Another important phenomenon is that of non-dispersive wave propagation. If a long slender rod, say steel, is impacted at one end, then, a disturbance will propagate along the rod at a fixed speed, and the shape of the disturbance will not alter. This idealisation is very close to measurements, except close to the impacted end. A torsional wave is even less dispersive. Dissipation due to internal friction is also of a very low order.

No dispersion and no dissipation mean that any disturbance will travel along the bar at a constant speed without change of form. That is, if the disturbance measured by the particle displacement, u, at a given time is ,say, part of a sine wave, then it will still be part of a sine wave at a later time t but displaced by a distance $x = ct$. We can express this mathematically as $u = f(x - ct)$, where f is any function, such as sine, and c is the speed of propagation.

If we consider the axial disturbance of the rod in axial impact, then, the second order wave equation is readily generated. This equation has the form of the second differential of the observed displacement, u, with respect to time, equated to a constant, times the second differential, with respect to distance along the rod,

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad .$$

This is seen to be true, since

$$\frac{\partial^2 u}{\partial t^2} = c^2 f''(x - ct) \quad \text{and} \quad \frac{\partial^2 u}{\partial x^2} = f''(x - ct)$$

where the double prime signifies the double differentiation of the function f with respect to the argument, $(x - ct)$.

The square root of this constant is the speed of propagation, c. If we now multiply all terms by the density, ρ, of the material, the

left hand side can be identified with the time rate of change of momentum per unit volume and the right hand side with spatial rate of change of strain. The constant ρc^2 is a property of the material.

$$\frac{\partial^2(\rho u)}{\partial t^2} = \rho c^2 \frac{\partial^2 u}{\partial x^2}$$

or
$$\frac{\partial}{\partial t}\left(\frac{\partial \rho u}{\partial t}\right) = \rho c^2 \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right).$$

Our experimenter now looks out at the rest of the universe and finds that the distant stars appear to be in fixed positions. At least, there appears to be no rotation relative to the preferred set of axes. There is no way in which it is possible to detect any translation relative to the stars so an absolute frame of reference cannot be defined.

Yet, a further experiment is made in which four equal objects are equally spaced in a given plane. These will be seen to drift towards a common centre as a result of their mutual gravity. To prevent this, light springs are interposed between opposite bodies. Both springs settle down with equal compression.

After some considerable time, it is noticed that relative to the original lengths one spring is longer and the other is shorter. On looking out into space, it is noticed that along the line of the longer spring there is a very large spherical object. Detailed measurements show that there is a relative acceleration between the spacecraft and the object. Repetition of all the previous experiments shows no variation, so it is concluded that the laws of motion are equally applicable in a free fall situation. The only measurable difference is in the behaviour of the four-body system: the larger the system, the larger the effect.

Imagine now that the spacecraft has managed to land on the large body. It is apparent that there is a considerable gravity force acting on all objects in the laboratory. Again, repeating all of the previous experiments show that by including the force of gravity all motion can calculated by the existing methods.

We have here developed the usual tools of dynamics from a different point of view, where we see that force is only introduced as a convenience. In this approach, force is introduced at a late stage; how one body knows of the presence of another body is a

question that does not need an immediate answer, no matter how strong is the desire to seek this knowledge.

The meaning of non-rotation of an inertial frame was left in abeyance, but the following observation may be made. A frame of reference which does not rotate with respect to the fixed stars is identical to a frame such that, in the absence of matter, light travels in straight lines. To proceed to special relativity, one needs to add that the speed of light is constant. It also follows from the above arguments that if force is a secondary concept, then, inertia and force fields are also secondary.

1.4 Reformulation of Newtonian dynamics and gravity

The law of gravity may be expressed without using the concept of force by relating the relative acceleration between two bodies to the sum of their masses and their relative positions. Mass, here, is simply a measure of the quantity of matter. The frame of reference is any frame that does not rotate with respect to the "fixed" stars. This is equivalent to stating that light rays, in the absence of matter, travel in straight lines. In the case of short duration impact of bodies of the same material, the observation is made that momentum is conserved. This statement is true for any frame of reference on the assumption that movement of the frame is negligible during the impact. It is now convenient to define force. The reason for its introduction is to simplify the relationship between different aspects of mechanics, such as elasticity and gravitation. Force is to dynamics as money is to commerce.

Newton states in his Principia [1] that weight is proportional to mass, and Einstein [4] comments on the equality of inertial mass to gravitational mass, adding that they are "so differently defined." The fact that the equality needed verification arose from treating force as a fundamental quantity. We have already stated that the view that force is a primary concept that was taken by D. Bernoulli and L. Euler [5], and that force is a derived concept was the opinion of H. Hertz [3] and J. d'Alembert [2].

1.5 Law of gravity

Making use of the ideas mentioned above, it is possible to define Newtonian gravity in a slightly different way. Choose a

frame of reference that is not rotating with respect to the stars. Consider now two isolated bodies, A and B, and postulate the law of gravity.

The acceleration of one body relative to the other is proportional to the sum of their masses and inversely proportional to the square of their separation. The direction is along the line joining the two bodies. The sense being such that the acceleration of A relative to B is towards B.

Let m be the mass, r the separation, and G the universal gravitational constant. Thus the acceleration of B relative to A is,

$$\boldsymbol{a}_{B/A} = -\frac{G(m_A + m_B)}{r_{B/A}^2}\boldsymbol{e}_{B/A} \qquad (1.1)$$

where $\boldsymbol{e}_{B/A}$ is the unit vector in the direction of B relative to A.

Because the kinematic terms are relative, the reference frame does not need to be a classical inertial frame. The frame may be accelerating relative to some arbitrary frame, but not rotating.

If the mass of B is very much less than the mass of A, then the relative acceleration becomes virtually independent of the mass of B and thereby independent of the material from which B is made.

Equation (1.1) can be used to calculate the mass of a body compared to a standard mass, since the total mass can be determined from measurements of relative acceleration and separation. Note that only one type of mass is defined. It is being assumed that the gravitational constant, G, is independent of the material of the mass.

The universal gravitational constant, G, is one of the important numbers in physics and has been the subject of experiment for over 200 years. In the usual Newtonian context, it is the constant in the equation expressing the inverse square law for gravitational force

$$F = -\frac{Gm_1m_2}{r^2}$$

or in the alternative view

$$a = -\frac{G(m_1 + m_2)}{r^2}$$ where a is the relative acceleration.

The first attempt to quantify G was made by the British physicist Henry Cavendish in 1798. The apparatus comprises two small masses, connected, as in a dumbbell, and suspended by a fibre with the axis horizontal. This is , then, placed symmetrically between two larger spheres, so that the gravitational attraction between the large and small masses gives a twist to the fibre. The stiffness of the fibre has previously been calibrated to give torque from measurements of the angle of twist. The main source of error with this arrangement is the reliability of the calibration. It is better to eliminate force from the argument. In recent experiments, [19] the twist in the fibre is minimised by rotating the apparatus about a vertical axis by adjusting the angular acceleration. The value of this acceleration allows G to be calculated, the value quoted is (6.674215 +/- 0.000092) $\times 10^{-11}$ m^3 kg^{-1} s^{-2}, an accuracy of 14 parts per million. In this set up, the dumbbell has been replaced by a flat plate, and the mass supporting structure is counter-rotated but given the same acceleration as the fibre support, so that the relative angular velocity between the plate and the mass is sensibly constant. The overall rotation helps to nullify effects due to other masses in the laboratory.

It has been assumed that the universal gravitational constant, G, is independent of the type of material. If we now look at how mass may be measured, it will transpire that this assumption is justified.

Looking at equation (1.1), consider m_A to be the standard mass to which every other mass will be compared. If $m_B << m_A$, then measurement of the relative acceleration at a known separation will give a value for G. If the masses are formed from the same material, then, further measurements of acceleration and separation will give the magnitude of the second mass. With both masses made from a second material, it is conceivable that a different value of G will be obtained.

Let the value of G vary with the type of material so that equation (1.1) may be written

$$\boldsymbol{a}_{B/A} = -\frac{\left(G_A m_A + G_B m_B\right)}{r_{B/A}^2}\boldsymbol{e}_{B/A} = -\frac{G_A\left(m_A + \left(G_B / G_A\right)m_B\right)}{r_{B/A}^2}\boldsymbol{e}_{B/A}$$

Using this equation to define the value of the mass of the body B, we see that the ratio of the gravitational constants can be

incorporated in the definition of mass; therefore, only one universal constant is needed.

When dealing with one material, say platinum, then the mass could be simply a count of the number of molecules times some convenient factor. So for a second material, e.g. carbon, it is still a number count but with a different factor, which then becomes part of the ratio of the gravitational constants.

If we measure the acceleration of an object falling to Earth, corrected for the Earth's rotation, we can substitute in the above equation and evaluate the mass of the Earth.

$$g = \frac{Gm_{earth}}{r_{earth}^2}$$

In this, we assume that the mass of the object dropped is small when compared to Earth. Knowing how the Earth plus Moon orbit the Sun, we can evaluate the mass of the Sun, again the mass of Earth plus Moon is negligible compared to that of the Sun. This can be checked by studying the motion of the other planets around the Sun.

The evaluation of the mass of the other planets can be done if there is a moon orbiting the planet. Where there is no moon, an estimate can be made studying the small perturbations caused to a planet's orbit, due to other planets. Recently, better results can be achieved by sending artificial satellites to orbit the planet. This has also been done for Earth.

It is convenient to define the centre of mass. Let r_G be the position of the centre of mass, referred to an arbitrary non-rotating frame, such that

$$m_A \boldsymbol{r}_A + m_B \boldsymbol{r}_B = (m_A + m_B)\boldsymbol{r}_G$$

or

$$m_A(\boldsymbol{r}_A - \boldsymbol{r}_G) + m_B(\boldsymbol{r}_B - \boldsymbol{r}_G) = 0$$

or

$$m_A \boldsymbol{r}_{A/G} + m_B \boldsymbol{r}_{B/G} = 0$$

Differentiating twice with respect to time gives

$$m_A \boldsymbol{a}_{A/G} + m_B \boldsymbol{a}_{B/G} = 0 \qquad (1.2)$$

where $\boldsymbol{a}$ is acceleration.